Simplified AC-Heated Probes Method[1]

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ABSTRACT

New AC-Heated Probes Method using two low-inertial probes (wire and strip of foil) is offered. The probes are immersed into liquid and heated by infrasonic frequency sine wave current. Projections of the signal on imaginary axis (quadrature parts) are registered at the same frequency as voltage fed the probes. With use of new simple asymptotic formulas it is possible to calculate directly the absolute values of thermal properties of liquids. Thermal effusivity is determined from value of signal from the foil probe and its area. Thermal conductivity is determined from the value of signal from the wire probe and its length. The received value of thermal effusivity is used to calculate correction for thermal conductivity value. Thermal diffusivity and volumetric specific heat then can be easily calculated. Ways of elimination of free convection influence are proposed.

KEY WORDS: liquids; measurement techniques, periodic heating, specific heat; thermal conductivity; thermal diffusivity; thermal effusivity.

1. INTRODUCTION

For the first time AC-Heated Wire Method was offered (to measure a group of four thermal properties - TP) by L.P.Filippov [1]. It was developed in Dissertations of S.N.Nefedov (1980), S.N.Kravchun (1983) and A.S.Tleoubaev (1987) on Physical faculty of Moscow Lomonosov State University. The method consists in registration of amplitude and phase of tripled frequency signal appearing on the bridge's diagonal in one of arms of which a low-inertial probe - thin wire - immersed into liquid is connected [2-6] whereas the bridge is fed by sine wave current. The method has the series of doubtless advantages comparing other methods, as follows:

- Very small thickness of the probed layer of liquid that makes the method especially effective at high temperatures because the obtained thermal conductivity are purely conductive ("molecular") one without radiative contribution.
- High informative outlet: a group of four TP and temperature are measured.
- Miniaturity and simplicity of measurement cell, that permits to use a very small quantities of liquid (few cubic cm and less) for measurements.
- Opportunity of total automatization of the measurement process and creation of installation (device) for realization of non-interrupted measurements (or control) in regime of scanning on temperature and pressure .

Despite of all these merits, the method is poorly widespread among researchers comparisons with Transient Hot Wire Method (see, for example [7]).

Retardation reasons, as appear, are:

- This method practically never was used for receipt of absolute values of TP, but only for relative measurements, referred to properties of reference substance - toluene.
- Misgivings that being present constant overheat of the probe and being reason of free convective flows can result in receipt of unreliable TP values [8].
- Intricate mathematical processing of received data as far as absence of formulas for direct calculation of TP. Properties should be computed by consecutive iterations with use of cylindrical Kelvin functions.

- Bulky measurement procedure related with necessity of registration of amplitude and phase of tripled frequency signal.

In the given work ways of overcoming the listed retardation reasons are offered. The simplification of the measurement procedure and installation is reached by registration only a projection of signal of main frequency on imaginary axis - so called quadrature signal - and by using of simple asymptotic formulas for calculations. Registration of the quadrature signal which brings information about probe's and liquids TP is considerably easier from technical point of view than registration of tripled frequency signal parameters. Also the quadrature signal practically does not sense the temperature drift of the cell that makes optional its strict thermostating.

2. THEORY

2.1. Solution of differential equation for temperature waves

When solving a problem of finding complex field of temperature waves the common thermal conductivity equation is reduced (for complex alternating component of temperature - $\tilde{T}$) to solution of wave differential equation being a special case of Helmholtz's one [9,10]:

$$\Delta \tilde{T} - (2\, i\omega / a)\, \tilde{T} = 0 \qquad (1)$$

where Δ - Laplace's operator; i - imaginary one; ω - cyclic frequency of voltage that feeds probe; a - thermal diffusivity of the probe's environmental. The given equation is a differential equation in partial derivative of second order and of elliptical type [9,10]

General solutions of this equation are:

1) in case of flat waves - linear combination of exponents:

$$\tilde{T} = A \exp\{ -ikx \} + B \exp\{ ikx \} \qquad (2)$$

where $k=(-2i\omega/a)^{1/2}$ - wave number of the temperature wave; x - distance from a plane of foil's center;

2) in case of cylindrical waves - linear combination of modified Bessel functions I_0 and K_0 or that of Kelvin functions $ber\kappa + i\,bei\kappa$ and $ker\kappa + i\,kei\kappa$:

$$\tilde{T} = A\cdot I_0(i^{1/2}\kappa) + B\cdot K_0(i^{1/2}\kappa) = A\cdot(ber\kappa + i\cdot bei\kappa) + B\cdot(ker\kappa + i\cdot kei\kappa) \qquad (3)$$

where κ - dimensionless thermal similarity parameter for the field of temperature waves - analog of $Fo^{-1/2}$ (Fo - Fourier number, r - radius coordinate):

$$\kappa = r\cdot(2\omega/a)^{1/2} \qquad (4)$$

First terms in these general solutions describe a wave the amplitude of which grows with increase of argument (x or r) - i.e. approaching wave. Correspondingly the second terms describe a wave the amplitude of which diminishes with increase of argument - i.e. moving away wave.

To find a special solution of a particular thermal problem it is required to define the complex constants A and B substituting these general solutions into following boundary conditions:

i) condition of probe's thermal balance - law of energy conservation;

ii) Sommerfeld's condition of attenuation of temperature wave at infinity [9];

iii) equality of temperatures and thermal flows at probe's surface - ideal thermal contact;

iv) some additional conditions, if needed for solving more difficult problems [6,11].

As a result, the complex alternating components of the probe's temperatures $\tilde{T}$ ($Re\,\tilde{T} > 0$, $Im\,\tilde{T} < 0$) are related with their reduced dimensionless complex temperatures $\tilde{\Theta}$ as follows:

$$\tilde{T} = [W/(4Cm\omega)]\cdot\tilde{\Theta}(\kappa,\eta) \qquad (5)$$

where W - amplitude of electrical power in probes; C - specific heat of the probe's material at constant pressure; m - mass of the probe; κ - thermal similarity parameter - see Eq. (4) -where in case of wire its radius is used and in case of foil its half-thickness h is used instead of the radius coordinate r; η - ratio of volumetric specific heats of probe's material and of environmental divided by 2:

$$\eta_w = C_w\rho_w/(2C\rho) \qquad (6)$$

$$\eta_f = C_f \rho_f /(2C\rho) \qquad (7)$$

- the subscripts w and f relate to wire and foil, respectively.

For probes located in vacuum κ is very small ~ 10^{-4}-10^{-5}, $\eta \to \infty$, $\kappa\eta \to \infty$, $\kappa^2\eta \to \infty$ and $\tilde{\Theta} = -i$. This with Eq.(5) can be used for calibration a value of $(dR/dT)/(Cm)$ to determine then a value of $\tilde{\Theta}$ from the measured $\tilde{T}$ value [2-5].

For probes located in liquid the following expressions for $\tilde{\Theta}$ were received:

1) in case of infinitely extended foil:

$$\tilde{\Theta}_f(\kappa_f \cdot \eta_f) = [\, i + i^{1/2} / (2\kappa_f \eta_f)]^{-1} \qquad (8)$$

2) in case of infinitely long wire:

$$\tilde{\Theta}_w(\kappa_w, \eta_w) = [\, i - (ker'\kappa_w + i \cdot kei'\kappa_w)/(ker\kappa_w + i \cdot kei\kappa_w)/(\kappa_f \cdot \eta_f)]^{-1} \qquad (9)$$

$ker\kappa$, $kei\kappa$, $ker'\kappa$, $kei'\kappa$ - Kelvin functions and their derivatives with respect to κ.

During deriving these equations (which are valid for all values of κ and η) it was assumed that temperature waves inside the probes are absent because its lengths:

$$l^* = 2\pi[a/(2\omega)]^{1/2} \qquad (10)$$

are much longer than the foil's thickness and the wire's diameter owing to that metals has much bigger thermal diffusivity than liquids.

2.2. Receipt of thermal effusivity absolute values by foil probe

To separate real and imaginary parts Eq.(8) can be re-written as

$$\tilde{\Theta}_f(\kappa_f \eta_f) = [2^{1/2}\kappa_f \eta_f - i \cdot \kappa_f \eta_f (\kappa_f \eta_f + 2^{1/2})] / [1 + 2 \cdot 2^{1/2}\kappa_f \eta_f + 4(\kappa_f \eta_f)^2] \qquad (11)$$

Using Eq.(5) it is possible to get a formula to determine an absolute value of thermal effusivity ε through measurement of imaginary component of signal from the foil probe (just as earlier through measurement of amplitude of tripled frequency signal [1])

$$\varepsilon = W / [4(2\omega)^{1/2} \cdot S_f \cdot Im\tilde{\Theta}_f] \cdot (1 + \delta_f) \qquad (12)$$

$$\delta_f \cong -4(\kappa_f \eta_f)^2 \qquad (13)$$

where S_f - area of the foil probe. Significance of the correction δ_f is very small (thickness of foil $2h$ is about 1-3 μm, so κ_f ~0 01-0.03, η ~1) and it is possible to be neglected, but if wanted it may be taken into account:

$$\kappa_f \eta_f \cong - [2 \cdot 2^{1/2} \, Im \tilde{\Theta}_f \; C_f \; m_f \, \omega] / W \qquad (14)$$

To ensure that the foil probe it was possible to be as a source of flat temperature waves its width (~1-2 mm) should be many times longer than a length of the temperature wave (~0.01-0.05 mm).

2.3. Approximate formulas for receipt of absolute TP values by the wire probe

New simple formulas for direct calculation of absolute TP values can be received with use of second order asymptotic formulas for Kelvin functions used in Eq.(9):

$$ker\kappa = -ln(\kappa\gamma/2) + (\pi/16)\cdot\kappa^2 + O(\kappa^4) \qquad (15)$$

$$kei\kappa = -\pi/4 + [1-ln(\kappa\gamma/2)]\cdot\kappa^2/4 + O(\kappa^4); \qquad (16)$$

$$ker'\kappa = -1/\kappa + (\pi/8)\cdot\kappa + O(\kappa^3) \qquad (17)$$

$$kei'\kappa = \kappa/4 - ln(\kappa\gamma/2)\cdot\kappa/2 + O(\kappa^3) \qquad (18)$$

(γ - Euler's constant equals to 1.781072418...)

Substituting these approximate expressions into Eq.(9) the reduced dimensionless complex temperature $\tilde{\Theta}$ of the wire probe is possible to be received:

$$\tilde{\Theta}_w(\kappa_w,\eta_w) = -\kappa^2_w\eta_w \, ln(\kappa_w\gamma/2)\,(1+\delta_1)/(1+\delta_3) - i\,(\pi/4)\,\kappa^2_w\eta_w\,(1+\delta_2)/(1+\delta_3) \qquad (19)$$

and consequently, substituting in Eq.(5) - formulas for direct TP calculation from results of measurements by the wire probe:

$$\lambda \cong -W/(16\,L_w\,Im\tilde{\Theta}_w)\cdot(1+\delta_2)/(1+\delta_3) \qquad (20)$$

$$a \cong r^2_w\gamma^2\omega/2\cdot exp\{-(\pi/2)\,(Re\tilde{\Theta}_w/Im\tilde{\Theta}_w)\,(1+\delta_2)/(1+\delta_1)\} \qquad (21)$$

here δ_1, δ_2, δ_3 - small corrections:

$$\delta_1 = -(\pi/8)\kappa^2_w/ln(\kappa_w\gamma/2) + O(\kappa^4_w); \qquad (22)$$

$$\delta_2 = (4/\pi)\kappa^2_w\eta_w\{[ln^2(\kappa_w\gamma/2)+\pi^2/16](1-0.5/\eta_w)+ln(\kappa_w\gamma/2)/2/\eta_w-0.25/\eta_w\}+O(\kappa^4_w) \qquad (23)$$

$$\delta_3 = (\pi/2)\, \kappa^2_w \eta_w\, (1-0.5/\eta_w) + O(\kappa^4_w); \tag{24}$$

These corrections δ_1, δ_2, δ_3, size of which at $\kappa < 0.3$ ($0.5 < \eta < 1.3$) does not exceed few percents, can be calculated with help of following expressions received from approximate formulas of the first order:

$$\kappa^2_w \eta_w \cong -16 C_w m_w \omega\, Im\tilde{\Theta}_w/(\pi W) \tag{25}$$

$$ln(\kappa_w \gamma/2) \cong (\pi/4)\, (Re\tilde{\Theta}_w / Im\tilde{\Theta}_w) \tag{26}$$

$$\kappa^2_w \cong (4/\gamma^2)\, exp\{(\pi/2)(Re\tilde{\Theta}_w / Im\tilde{\Theta}_w)\} \tag{27}$$

$$\eta_w = \kappa^2_w \eta_w / \kappa^2_w \tag{28}$$

As evident from the Eq. (19) to receipt absolute value of thermal conductivity it is sufficient to measure imaginary component of the wire probe signal. The knowledge of probe's length L_w is required. To receipt thermal diffusivity absolute value from the Eq.(21) it is sufficient to measure a phase (or ratio of synphase and quadrature components) of the wire's temperature oscillations and frequency ω. Knowledge of wire probe's radius is required. Besides, of course, values of power supply W and of derivative dR/dT are needed.

Application of these asymptotic Eqs.(15-27) is lawful if one use sufficiently thin wires and not very high frequencies of feeding voltage for k to be not bigger than ~0.3. For example, for a wire of 12.7 μm diameter at frequency of feeding voltage of 5 Hz and typical value of thermal diffusivity of liquid ~$9 \cdot 10^{-8}$ $m^2 \cdot s^{-1}$ (toluene at normal conditions) $\kappa = 0.168$.

Computer calculations have shown (see Fig.1.), that deviations of values $Im\tilde{\Theta}$ and $Im\tilde{\Theta}/Re\tilde{\Theta}$, calculated with approximate Eqs.(19,22,23,24) comparing these functions calculated with exact Eq.(9) does not exceed 0.2 % at $\kappa < 0.3$ (calculations were made for values of η from 0.6 to 1.2 - typical values for liquids at normal conditions and platinum probe).

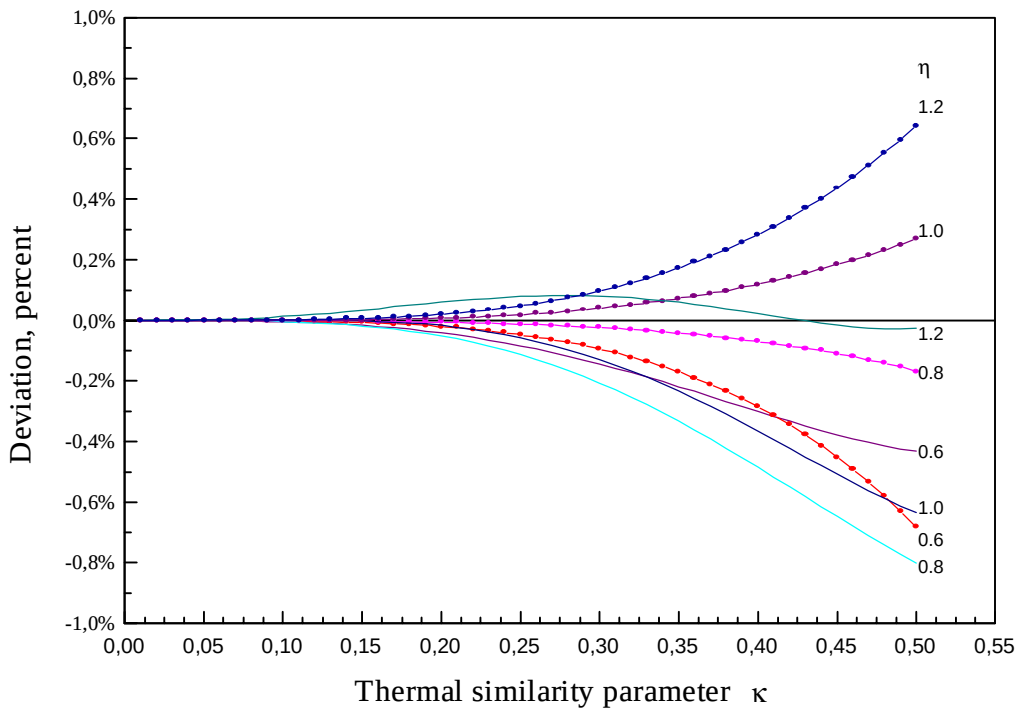

Fig. 1. Deviations of values of $Im\tilde{\Theta}$ (smooth lines) and $Im\tilde{\Theta}/Re\tilde{\Theta}$ (rough lines) calculated with help of approximate Eqs.(19, 22,23,24) comparing with exact Eq(9) vs. κ for some values of η.

Measurements of $Re\tilde{\Theta}$, (and correspondingly of ratio $Im\tilde{\Theta}/Re\tilde{\Theta}$) with good accuracy is possible only at registration on the *tripled* frequency because a value of $Re\tilde{\Theta}$ at the *main* frequency is merged with synphase signal of non-ideality of bridge balancing (which is a subject to strong influence of cell's temperature drift). In following section will be shown how it is possible to measure group of four TP without need of registration of tripled frequency signal parameters.

2.4. Use of two probes for receipt of group of four TP

With registration of values of $Im\tilde{\Theta}$ from wire and foil probes on the main frequency ω, the absolute values of group of four TP successively appears possible to be received on following procedure:

i) Value of thermal effusivity ε from measured value of $Im\tilde{\Theta}_f$ by Eq.(12).

ii) Approximate values of thermal conductivity λ and product $\kappa^2\eta$ from measured value of $Im\tilde{\Theta}_w$ by Eqs. (20) and (25).

iii) Approximate value of thermal diffusivity by formula:

$$a=\lambda^2/\varepsilon^2 \tag{29}$$

and then approximate value of parameter κ by Eq.(4) (value of the wire's radius r_w is approximately known)

iv) Approximate value of parameter η_w by formula :

$$\eta_w=(\kappa^2_w\eta_w\cdot a)/(r^2_w\cdot 2\omega) \tag{30}$$

v) Values of corrections δ_2 and δ_3 by Eqs.(23,24) and exact value of thermal conductivity λ by Eq(20).

vi) At last, exact values of two other TP - thermal diffusivity a on Eq. (29) and volumetric specific heat $C\rho$ by formula:

$$C\rho=\varepsilon^2/\lambda \tag{31}$$

Another more convenient way of receiving exact value of thermal conductivity λ (instead of item v) is through the use of graph on Fig.2 where the value of correction for approximate thermal conductivity value versus parameter κ for various values of parameter η is plotted. As far as the correction is small (it does not exceed few percents) accuracy of this graph is quite acceptable.

In general, if only thermal conductivity measurements using only wire probe are carried out then handbook data for calculation of approximate parameter η can be used and parameter κ can result through Eq. (25) to use graph on Fig.2 to determine the value of correction for λ.

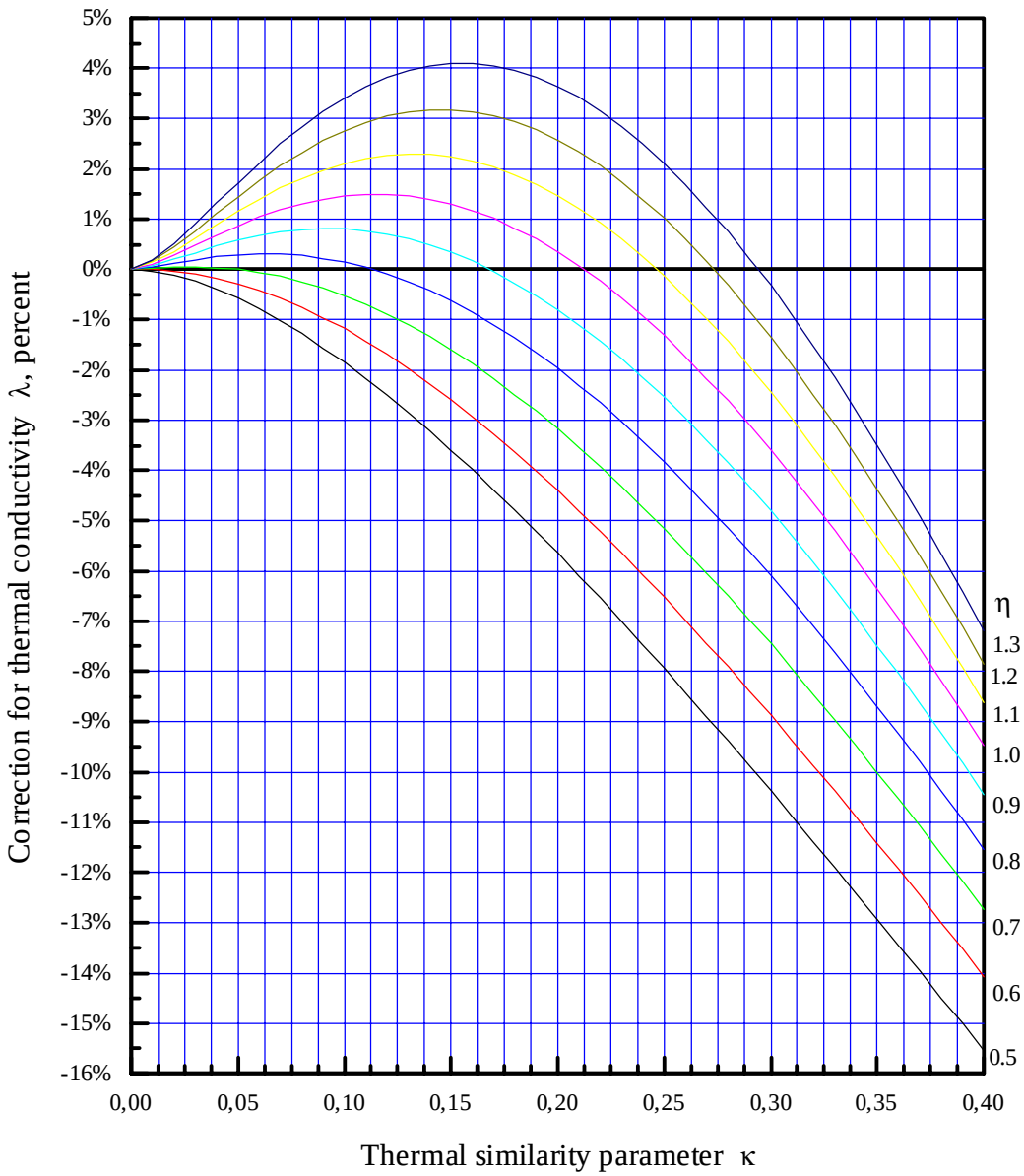

Fig. 2. Correction for thermal conductivity calculated by exact Eqs.(9,5) vs. κ for some values of η.

3. EXCLUSION OF FREE CONVECTIVE FLOWS INFLUENCE

In the method of AC-Heated Probes free convective flows can take place because of probe's constant overheating concerning to cell walls. Thermal boundary layer thickness δ at free convection can be estimated as [12] (if the probe is vertical):

$$\delta \cong [(4\nu^2 x)/(g\,\beta'\,T)]^{1/4} \qquad (32)$$

where ν - kinematic viscosity of the liquid, x - distance from the lower end of the probe, g - 9.8 m·s^{-2}, β' - factor of liquid's thermal expansion, T - probe's overheating.

Significance of δ for toluene at 473K and at overheat T=4K is about 0.1 mm already at x = 0.032 mm and then increases as $x^{1/4}$. At the same time the length of temperature wave is about 0.01-0.05 mm, and owing to that the temperature wave is strongly attenuates (on length of wave in $exp\{-2\pi\}$ ~500 times) so only motionless boundary layer of liquid adjacent to the probes surfaces is probed during measurements.

As a further checking a thermal problem assuming the equality to zero the temperature oscillations $\tilde{T}$ on some distance δ from the wire's surface (for instance owing to flow of liquid in parallel of probe's axis) was analytically solved. Computations on basis of the received solution showed that in case if the length of temperature wave is less than δ (thickness of boundary layer of liquid) then the flow of liquid practically does not influences to wire's temperature oscillations.

The most simple and reliable way to exclude the distorting influence of convective flows is the greatest possible reduction of the cell's diameter. Within the cell of small diameter convective flows are practically absent, that guarantees the receipt of undistorted TP values. As was experimentally shown in [13] slow flows does not influence to the probe's temperature oscillations at Reynolds numbers up to 150 and at flow velocities up to few cm·s^{-1} (at frequency 23 Hz).

On basis of above-stated reasons, and also being present experience the best solution for the cell construction is a probe, tightened along axis of a tube of 3-5 mm ID. Tension created through small spring or weight is necessary as far as owing to

probe's lengthening the amplitude of cross oscillations in the middle of the probe is equal to:

$$L_w(\alpha' \mid \tilde{T} \mid /2)^{1/2} \qquad (33)$$

(α'- temperature factor of linear extension - for platinum equals ~$9 \cdot 10^{-6}$ K^{-1}) in case of non-tightened probe can exceed in many times the wire's diameter (12-20 μm) that is not permissible. Trial approaches to measure TP of reference liquids - toluene and carbon tetrachloride - using non-tightened sagged probes almost always resulted in receipt of distorted TP values.

5. CONCLUSIONS

New simplified AC-Heated Probes Method with direct calculation formulas for measurements of thermal conductivity, thermal effusivity, thermal diffusivity and volumetric specific heat of liquids with use of two low-inertial probes - wire and foil - and registration of quadrature signal on frequency of feeding voltage is developed. Installation based on the method can be assembled with use of serially produced vector generator and lock-in microvoltmeter. Description of the installation, its error analysis and testing results soon will be presented in coauthorship with O.L.Kotlyarov.

This method and formulas also can become a base for a new type of intelligent transducers for non-interrupted measurement and/or control of TP in chemical engineering processes.

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FIGURE CAPTIONS

Fig. 1. Deviations of values of $Im\tilde{\Theta}$ (smooth lines) and $Im\tilde{\Theta}/Re\tilde{\Theta}$ (rough lines) calculated with help of approximate Eqs.(19, 22,23,24) comparing with exact Eq(9) vs. κ for some values of η.

Fig. 2. **C**orrection for thermal conductivity calculated by exact Eqs.(9,5) vs. κ for some values of η.

LIST OF SYMBOLS

a thermal diffusivity $a=\lambda/C\rho$

$ber\kappa$, $bei\kappa$, $ber'\kappa$, $bei'\kappa$ Kelvin functions and their derivatives

C specific heat at constant pressure

$C\rho$ volumetric specific heat

g gravitational acceleration

h half-thickness of the foil probe

I_0, I_1 modified Bessel functions

i imaginary one

K_0, K_1 modified Bessel functions

$ker\kappa$, $kei\kappa$, $ker'\kappa$, $kei'\kappa$ Kelvin functions and their derivatives

L length of the probe

l^* length of temperature wave $l^*= 2\pi[a/(2\omega)]^{1/2}$

m mass of the probe

r radius

S foil probe's area

T probe's overheat

$\tilde{T}$ complex alternating component of the probe's temperature

W amplitude of power supply in the probe

x distance from a plane of foil's center

GREEK SYMBOLS

α' temperature factor of probe's linear extension

β' temperature factor of liquid's volume expansion

γ Euler's constant γ=1.7810724...

Δ Laplace's operator

δ temperature boundary layer thickness

δ_1, δ_2, δ_3, δ_f small dimensionless corrections

ε thermal effusivity, $\varepsilon = (\lambda C\rho)^{1/2}$

η ratio of volumetric specific heats of probe's material and liquid divided by 2:

$\eta_w = C_w\rho_w/(C\rho)$, $\eta_f = C_f\rho_f/(C\rho)$.

κ thermal similarity parameter: $r_w(2\omega/a)^{1/2}$ for the wire and $h_f(2\omega/a)^{1/2}$ for the foil

λ thermal conductivity

ν kinematic viscosity

ρ density

$\tilde{\Theta}$ reduced dimensionless complex temperature of the probes

ω circular frequency of the voltage

SUBSCRIPTS

f referred to the foil probe

w referred to the wire probe