Percolation Transition, Stipulated by Generation of Ice in Small Volumes of Cellular Bio-Structures[1]

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1. ABSTRACT

The model of a biological system is proposed. This system consists of two phases: hard and soft. It is established the percolation transition must take place in this model. The theoretical dependencies of the shear modulus on the hard phase concentration and the temperature were received.

The mechanism of the hard phase forming is coordinated with the biological mechanism the ice forming. It is showed how calculate the ice quantity in the system using the shear modulus values. The elastic modulus of the sugar-beet are measured in the temperature interval 210-270 K. The percolation transition is observed which was predicted by the used model. The ice quantity in the investigated sugar-beet is calculated for the all temperature from the researched interval.

KEY WORDS: biological systems, elastic modulus, liquid-solid transition, low temperature, percolation, sugar-beet.

1. INTRODUCTION

Termophysical properties of biological systems are investigated in the given article. It is known the water is the great part of biological systems. The decreasing the temperature must lead to the liquid-solid transition there. We want to research its peculiarities.

Denote by m the relative volume occupied with the ice (the concentration of the ice).

The common quantity of the ice in the biological systems is determined by the calorimeter method [1]. We want to find the dependence m on the temperature T. For this aim we suppose to use the dependence of the shear modulus G on the temperature. It is necessary find the connection between G and m.

2. THE TWO PHASES MODEL OF THE BIOLOGICAL TISSUE

Let us consider the material of system as a continuum. Denote by g(x) a local modulus in the point x of the continuum. Let us g(x) be an accidental value.

Also denote by the K(x,x') the correlation function which is determined by the formula

$$K(x,x') = \langle g(x)g(x')\rangle - \langle g\rangle^2, \tag{1}$$

where the average values are denoted with the help of the operator $\langle \ldots \rangle$.

The measured modulus G of the systems is the effective modulus. One is also some average value of g

$$G = \overline{g} \tag{2}$$

Generally speaking the $\overline{g}$ isn't equal to the $\langle g \rangle$.

The characteristic linear size is the correlation radius a determined by the formulas

$$K(x,x') \neq 0 \qquad |x - x'| < a$$
$$K(x,x') = 0 \qquad |x - x'| > a \tag{3}$$

We will suppose that "our" continuum consist of two phases: hard and soft. Let us accept the notations G_h and G_s for the shear modulus of this phases. Then if the hard or soft phase is in the point x the formulas g(x)= G_h or g(x)=G_s take place.

We also will suppose that the hard phase forms the clusters which are surrounded by the soft phase. The size of the cluster is equal to the correlation radius in the order of the value.

3. PERCOLATION TRANSITION IN THE TWO PHASES MODEL

When the concentration of the hard phase is increasing the correlation radius a is increasing also. By some concentration $n=n_0$ of the hard phase the correlation radius a reaches the size which is equal to the size of the system in the order of the value. The hard phase forms the framework which accepts the external loading. By this the modulus G increases strongly.

Denote by G_0 the modulus value by $n<n_0$. If the inequality

$$g_s << g_n \tag{4}$$

is fulfilled the conditions are fair

$$G_0 \approx 0 \qquad (n < n_0) \tag{5}$$

$$G >> G_0 \qquad (n \geq n_0) \tag{6}$$

The dependence G(n) have the jump by the concentration n_0. As they say the percolation transition is observed by this concentration. It is known from the percolation theory [2] for the accidental medium the value of n_0 is equal to 0.15.Also it is known from this theory the dependence G(n) have the form

$$G(n) = b(n - n_0)^t, \tag{7}$$

where b and t are the constant values. Taking into consideration the condition

$$G \rightarrow g_n \qquad (n \rightarrow 1) \tag{8}$$

instead of the formula (7) we can write

$$n = n_0 + (1 - n_0)\left(\frac{G}{g_h}\right)^{1/t} \tag{9}$$

4. THE MODULUS DEPENDENCE OF THE TWO PHASES MODEL ON THE TEMPERATURE

The concentration n is the function of the temperature T. Therefore the temperature T_0 must be for which the conditions is fulfilled

$$n < n_0 \qquad (T > T_0) \tag{10}$$

$$n \geq n_0 \qquad (T \leq T_0) \tag{11}$$

The value n_0 is reached by the temperature T_0. Accordingly the dependence G(T) must have the form

$$G=G_0\approx 0 \qquad (T>T_0) \tag{12}$$

$$G>>G_0 \qquad (T\leq T_0) \tag{13}$$

Expand the function $n-n_0$ in the series about $T-T_0$ keeping only the linear term

$$n-n_0=\left.\frac{\partial n}{\partial T}\right|_{T=T_0}\left(T-T_0\right) \tag{14}$$

Substituting (14) in (7) we receive

$$G=Q(T-T_0)^t, \tag{15}$$

where Q is $b\left.\frac{dn}{dT}\right|_{T=T_0}$.

Using the formula (15) it is possible to get the value t from the experimental dependence G(T).

5. THE QUANTITY OF THE ICE IN THE CELLULAR TISSUE

There is the formula (9) which allow to calculate the quantity of the hard phase. Only it is necessary to know the numerical value of g_h. What is the hard phase in the cellular tissue?

The mechanism of forming an ice in the cellular tissue is proposed in [3].

Forming the ice in the cell means it's ruin. The alive organism aspires to avoid it pushing out the water from the cell in the space between the cells where the water is frozen. The structure arises which consist of the cells surrounded by the layer of the ice. The liquid matter is inside the cells.

We call such structure as the hard phase of the our model. In this structure the ice occupies only a part x of the volume. One can write

$$m=x_n \tag{16}$$

Let us find the numerical value of x. Suppose that the cell have the cubic form. Denote by L and h the cell size and the thickness of the ice layer. The volume which occupies the ice is equal to $6L^2h/2$ on one cell and we receive

$$x=3h/L \tag{17}$$

The modulus g_h is the average modulus of the hard phase. The liquid mater inside the cells have very small modulus comparing with the ice. Therefore we write

$$g_h=xg_i \tag{18}$$

where g_i is the modulus of the ice. According to the literature data $g_i=3.5\cdot10^9$ Pa. Taking $h\approx0.1$ L we receive x=0.3, and $g_h=1.2\cdot10^9$Pa. Using this numerical value of g_h, the formulas (9) and (16) and the experimental curve G(t) we may build the searched dependence m(T).

6. MEASUREMENTS AND EXPERIMENTAL RESULTS

As the example of the biological system a tissue of a sugar-beet was chosen. The shear modulus of this specimens was measured with the help of the forsion pendulum [4]. The error was 4%. The frequency of the vibration was 0.01 Hz. The experimental dependence G(t) is given in fig.1.

It is clear this dependence must be described by the conditions (12-13). This shows that the chosen model is fair.

Using the experimental curve G(T) and the formula (15) we find the value t, which is equal to 1.48±0.02.

Also using the experimental curve G(t) and the formulas (9) and (16) we build the dependence m(T) for the tissue of a sugar-beet (fig. 2). The value of the common ice quantity calculated from the modulus values is equal to one received in the work [1].

7. CONCLUSIONS

In given article it is predicted theoretically that the percolation transition must be in the biological system by decreasing the temperature. This prophesy is confirmed experimentally for the sugar-beet. The common character of the used theoretical model allow to hope that such the transition will be observed for other biological systems. Accordingly the proposed method of the calculating of the ice quantity will be fair in this cases.

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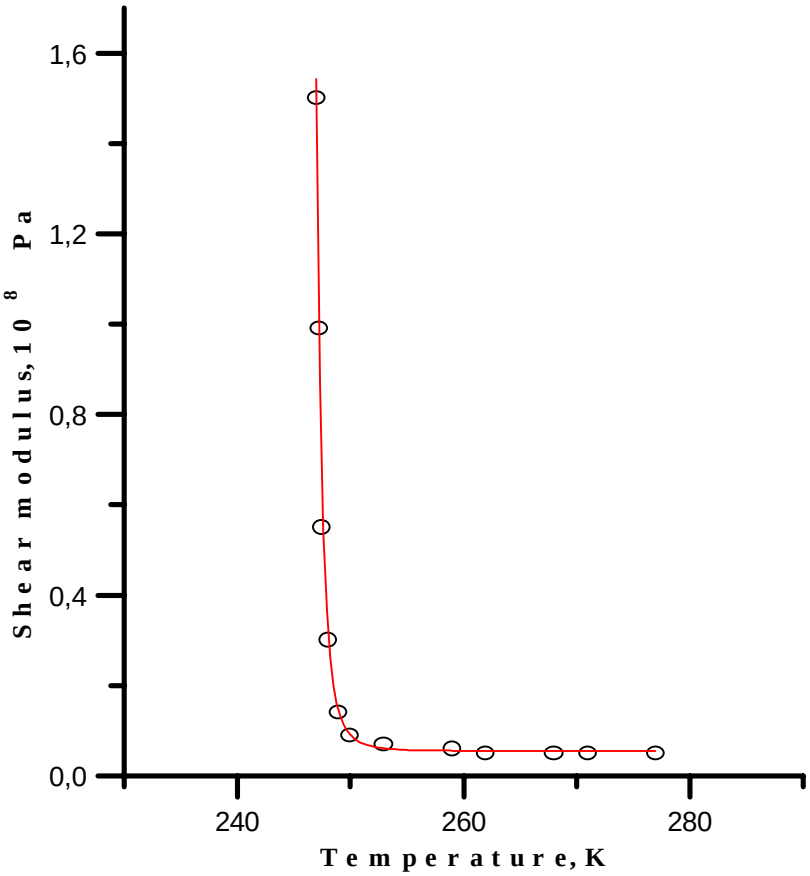
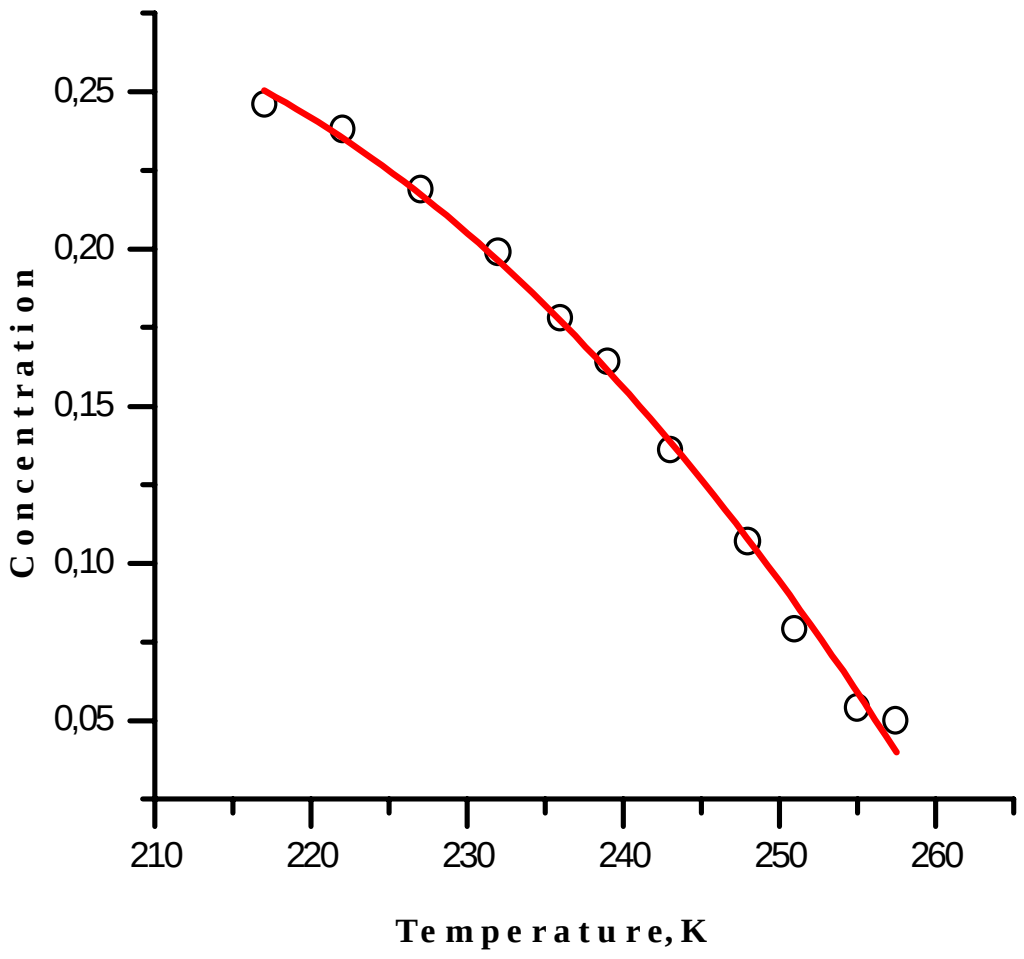
FIGURE CAPTIONS

Fig.1. The temperature dependece of the shear modulus in the sugar-beet tissue.

Fig.2. The temperature dependece of the ice concentration in the sugar-beet tissue.




0,25
0,20
0,15
0,10
0,05
Concentration
210
220
230
240
250
260
Temperature, K