Heat Conduction Very Near the Superfluid Transition in ^{4}He[1]

W. A. Moeur[2,3], P. K. Day[4], F-C Liu[4], and R. V. Duncan[2]

---

[1] Paper presented at the Thirteenth Symposium on Thermophysical Properties, June 22-27, 1997, Boulder, CO.

[2] Department of Physics and Astronomy, University of new Mexico, Albuquerque, NM.

[3] To whom correspondence should be addressed.

[4] Jet Propulsion Laboratory, Pasadena, CA.

Abstract

We report on an experimental observation of self-organized criticality in ^{4}He very close to its superfluid transition. A constant temperature gradient, independent of the heat flux Q through the sample, is created along a vertical column of ^{4}He by applying heat to the top of the column. This constant temperature gradient equals the gravity-induced gradient in the superfluid transition temperature, indicating that the thermal conductivity of the sample has self-organized. The closeness to criticality in this state is the same throughout most of the sample, and it depends only on Q. These measurements have been made in a range of Q from 0.04 to 6.5 mW/cm^2 in the absence of convection.

This work was accomplished with support from the NASA Microgravity Science and Applications Division. This ground-based work supports a flight project called Critical Dynamics in Microgravity (DYNAMX) which intends to measure critical heat transport through ^{4}He very close to the superfluid transition in a microgravity environment.

KEY WORDS: critical phenomena; self-organized criticality; superfluid; thermal conductivity; thermal transport properties.

1. Introduction

In this paper we report on an observation of self-organized criticality (SOC) in the thermal conductivity κ of ^{4}He near its superfluid transition. Self-organization has been observed in several other physical systems near criticality, and is thought to be important in a wide realm of phenomena [1]. While other observations of SOC have occurred in systems which exhibit hysteretic or 'avalanche' behavior [2], the self-organization reported here occurs near the continuous, non-hysteretic phase transition to superfluidity in ^{4}He [3]. We find that the self-organized state in ^{4}He facilitates a new technique for non-equilibrium measurements very close to the superfluid transition. The nature of this self-organization may be understood by employing a simple model which relates the temperature at which the system self-organizes, to the thermal conductivity of ^{4}He and the depression of the transition temperature due to a heat flux.

The study of transport properties close to a continuous phase transition is difficult because the out-of-equilibrium situation required to observe the properties generates gradients in the properties themselves. This inhomogeneous situation rounds the sharpness of the transition under study and limits the accuracy with which the measurements may be made [4]. Such problems are reduced in systems which support transport without a gradient in the associated thermodynamic potential. Examples include ^{3}He and ^{4}He,

which allow heat transport without an associated temperature gradient in their superfluid phase. In the normal phase the gradient appears, making conclusive measurements of transport properties through the transition difficult. A second inhomogeneity is created by Earth's gravity which produces a static pressure gradient, and hence a gradient in the superfluid transition temperature, across the sample [5].

Onuki predicts that these two different sources of inhomogeneity can effectively offset one another, causing the helium to remain at a constant distance from criticality within its normal phase [6]. This homogeneity permits heat transport measurements much closer to criticality than is otherwise possible in the presence of temperature and pressure gradients. This is one specific example of self-organization, which is possible in systems with a divergent thermal diffusivity, as discussed by Carlson et al. [7]. Machta et al. [8] applied this theory to ^{4}He and predicted that it should self-organize near its superfluid transition. Recently, Ahlers and Liu [9] determined the conditions which would permit the experimental observation of this self-organization.

2. Heating From Below

Typically, thermal conductivity measurements in helium have been performed by heating a vertical column of helium from the bottom. Consider the heat flow equation for this situation.

$$\nabla T = \frac{-|Q|}{k(\tau)} \tag{2.1}$$

where,

$$\tau = \frac{T(z) - T_c(Q,z)}{T_{\lambda o}} \tag{2.2}$$

here, $T_c(Q,z)$ is the Q-dependent superfluid transition temperature [10]. Note that $T_c(Q = 0,z) = T_\lambda(z) = T_{\lambda o} + \nabla T_\lambda$, where $\nabla T_\lambda = 1.273$ μK/cm [5] and $T_{\lambda o} = 2.1768$ K.

then,

$$\nabla\tau = \frac{\nabla T - \nabla T_\lambda}{T_{\lambda o}} \tag{2.3}$$

so that

$$T_{\lambda o}\nabla\tau = \frac{-|Q|}{k(\tau)} - \nabla T_\lambda \tag{2.4}$$

This says that $\nabla\tau$ is always negative, or that as you travel downward from the interface, τ is always increasing.

Figure 1 shows a phase diagram for a cell that has heat applied to the bottom, next to a schematic representation of the cell. The temperature on the top of the cell is colder than that on the bottom. Notice that after crossing the λ-line, the temperature profiles move away from it.

3. Heating From Above

Consider now a situation where the heat is applied to the top of the column of helium [11]. Then the sign of Q changes and equation (2.4) becomes:

$$T_{l\,o}\nabla t = \frac{|Q|}{k(t)} - \nabla T_l \tag{3.1}$$

This sign change allows $\nabla\tau$ to go to zero when κ reaches the value

$$k(t_{soc}) = \frac{|Q|}{\nabla T_l} \tag{3.2}$$

or when

$$\nabla T = \nabla T_l \tag{3.3}$$

Equation (3.2) implies that the thermal conductivity is constant throughout the self-organized region and Eqn. (3.3) says that the reduced temperature maintains a constant value, $\tau_{soc} = [T_{soc}(z) - T_c(Q,z)]/T_{\lambda o}$, throughout this region as well.

Figure 2 depicts a column of helium with an interface somewhere in the middle. Since heat is applied to the top of the column, the temperature on the top of the cell is higher than the temperature on the bottom. Traveling downward from the interface, τ increases with a decreasing slope until it achieves a reduced temperature of τ_{soc}, when $\nabla\tau$ becomes zero and τ stops changing.

τ_{soc} is the unique reduced temperature that gives a value for thermal conductivity that satisfies Eqn (3.2). The phase diagram to the right of the cell

shows two temperature profiles for different values of heat flux. Notice how the slope of the temperature profiles is now driven toward the slope of the λ-line. Once the slope of the temperature profile equals the slope of the λ-line, the system is said to have self-organized at a constant distance from criticality.

4. The SOC State Temperature

In the SOC state, The distance from criticality, τ_{soc}, is dependent only on the quantity of heat flux through the sample. It is reasonable to assume that over a small range in τ, $\kappa(\tau)$ can be approximated by,

$$\kappa(\tau) = \frac{\kappa_o}{\tau^x} \tag{4.1}$$

Inserting this expression into Eqn (3.2) and inverting, gives an expression for τ_{soc},

$$\tau_{soc}(Q) = \left(\frac{\kappa_o \nabla T_\lambda}{|Q|}\right)^{\frac{1}{x}} \tag{4.2}$$

and thereby the SOC state temperature T_{soc},

$$T_{soc}(Q) = T_c(Q) + \Delta T_{soc}(Q) \tag{4.3}$$

where

$$\Delta T_{soc}(Q) = T_{\lambda o}\left(\frac{\kappa_o \nabla T_\lambda}{Q}\right)^{\frac{1}{x}} \tag{4.4}$$

is the difference between $T_{soc}(Q)$ and $T_c(Q)$.

5. SOC Data

Figure 3 shows $T_{soc}(Q)$ measurements made for a range of heat flux $0.04 \leq Q \leq 6.5$ μW/cm². Since it is difficult to precisely measure $T_{l\,o}$, each measurement was instead referenced to T_{soc} at Q_{ref} = 100 nW/cm², so what is plotted in the figure is $\delta T_{soc} = T_{soc}(Q) - T_{soc}(Q_{ref})$.

For values of Q > 200 nW/cm², $\Delta T_{soc}(Q)$ is less than our thermometer resolution, so for this range of Q, $T_{soc}(Q) \approx T_c(Q)$. Measurements of T_{soc} in this range, provide a convenient and accurate way to measure the depression of T_c with heat flux.

6. Measurements of $T_c(Q)$

Previous measurements of the depression of the transition temperature with heat flux [10] provide a phenomenological equation for $T_c(Q)$,

$$T_c(Q) = T_{l\,o} - T_{l\,o}\left|Q/Q_o\right|^y \tag{6.1}$$

Since our data,

$$\mathrm{d}T_{soc} = T_{soc}(Q) - T_{soc}(Q_{ref}) \tag{6.2}$$

can be written

$$\mathrm{d}T_{soc} = T_c(Q) - T_c(Q_{ref}) + \Delta T_{soc}(Q) - \Delta T_{soc}(Q_{ref}) \tag{6.3}$$

it can also be represented by

$$\delta T_{soc} = \Theta - T_{\lambda o}\left(\frac{Q}{Q_o}\right)^y + \Delta T_{soc}(Q) \tag{6.4}$$

where

$$\Theta = T_{\lambda o}\left(\frac{Q_{ref}}{Q_o}\right)^y - \Delta T_{soc}(Q_{ref}) \tag{6.5}$$

is a constant and $\Delta T_{soc}(Q) \approx 0$ for the higher values of Q. With this in mind, the shift in T_c can be written as $T_{\lambda o}$ - $T_c(Q)$ = Θ - δT_{soc} which is plotted in Figure 4. The solid line in the figure is a fit of the data over the range 0.2 $\mu W/cm^2 \leq Q < 6.5\ \mu W/cm^2$ to Eqn. (6.1) and gives values of Q_o = 699 ± 196 W/cm^2 and y = .809 ± .012. This fit agrees closely with data taken by Duncan, Ahlers, and Steinburg [**Error! Bookmark not defined.**] in 1988 for values of heat flux $0.4 \leq Q \leq 10\ \mu W/cm^2$ in a heat from below geometry.

7. Thermal Conductivity

In the SOC state, the thermal conductivity is only a function of the applied heat current, so it is a known quantity:

$$k_{soc}(Q) = \frac{|Q|}{\nabla T_\lambda}. \tag{7.1}$$

What isn't known then, is the SOC temperature at which this conductivity exists. The measurements we've made can be used to determine this. Our data as taken is,

$$\mathrm{d}T_{soc}(Q) = T_{soc}(Q) - T_{soc}(Q_{ref}) \tag{7.2}$$

So that,

$$\mathrm{d}T_{soc}(Q) - \mathrm{d}T_{soc}(Q_{high}) = T_{soc}(Q) - T_{soc}(Q_{high}) \tag{7.3}$$

or

$$\mathrm{d}T_{soc}(Q) - \mathrm{d}T_{soc}(Q_{high}) = T_{soc}(Q) - T_{\mathrm{l}}(Q_{high}) \tag{7.4}$$

The reduced temperature is then,

$$\mathrm{t}_{soc} = \frac{T_{soc}(Q) - T_c(Q)}{T_{\mathrm{l}\,o}} = \frac{\mathrm{d}T_{soc}(Q) - \mathrm{d}T_{soc}(Q_{high}) + T_c(Q_{high}) - T_c(Q)}{T_{\mathrm{l}\,o}} \tag{7.5}$$

where the value of $T_c(Q_{\mathrm{high}})$ is known from previous measurements and an extrapolation of the same can give a value for $T_c(Q)$.

Figure 5 shows the SOC data displayed in this manner. In this region of temperatures, over this range of Q, the thermal conductivity is predicted to depend strongly on Q, [12].so it is not possible to compile a complete mapping of κ as a function of temperature for different values of Q.

References

[1] Per Bak, how nature works (Copernicus, Springer Verlag, New York, 1996).

[2] P. Bak, C. Tang, and K. Wiesenfeld, Phys. Rev. A **38**, 364 (1988).

[3] F. C. Liu and Guenter Ahlers, Physica B, **194-196**, 597 (1994).

[4] See, for instance Guenter Ahlers and Robert Duncan in Frontiers of Physics, Proceedings of the Landau Memorial Conference, E.Gotsman,Y. Ne'eman, and A. Voronel (eds.), p.219 (Oxford : Pergamon Press 1990).

[5] G. Ahlers, Phys. Rev. **171**, 275 (1968).

[6] A. Onuki, Proc. of the Fifth Oregon Conference on Low Temperature Physics, Eugene, OR, 132, (February 1-3, 1987).

[7] J. M. Carlson, J. T. Chayes, E. R. Grannan, and G. H. Swindle, Phys. Rev. Lett. **65**, 2547 (1990).

[8] J. Machta, D. Candela, R. B. Hallock, Phys. Rev. B **47**, 4581 (1988).

[9] G. Ahlers and F-C. Liu, J. Low Temp. Phys. **105**, 255 (1996).

[10] R. Duncan, G. Ahlers and V. Steinberg, Phys. Rev. Lett. **60**, 1522, (1988).

[11] Concerns about convection are addressed in G. Ahlers and F-C Liu, J. Low Temp. Phys., **105**, 255 (1996).

[12] R. Haussmann and V. Dohm, Phys. Rev. Lett. **77**, 980, (1996)

Figure Captions

Figure 1 Temperature profile which results when heat is applied to the bottom of a sample of helium.

Figure 2 Temperature profile that comes about by heating a sample of helium from the top.

Figure 3 SOC state temperature vs. applied heat flux. The dashed line is a plot of $T_c(Q)$ according to Eqn. (6.1)

Figure 4 Measurements of $T_c(Q)$ using the SOC state. The solid line is a fit to Eqn. (6.1).

Figure 5 Plot of SOC state thermal conductivity vs. SOC state reduced temperature. The numbers next to each data point represent the values of Q applied in nW/cm^2.

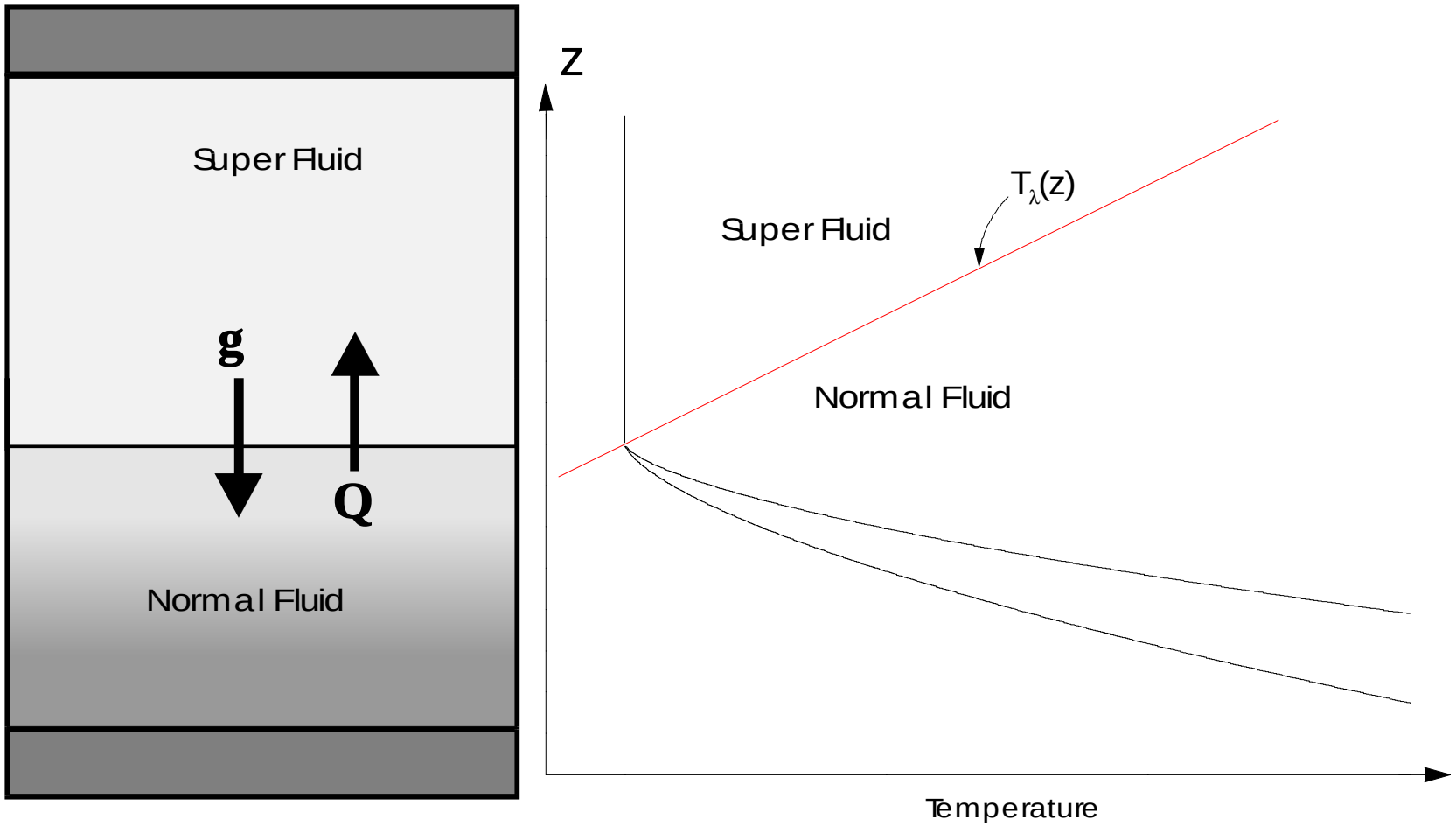
Super Fluid
g
Q
Normal Fluid
Z
$T_\lambda(z)$
Super Fluid
Normal Fluid
Temperature

Figure 1

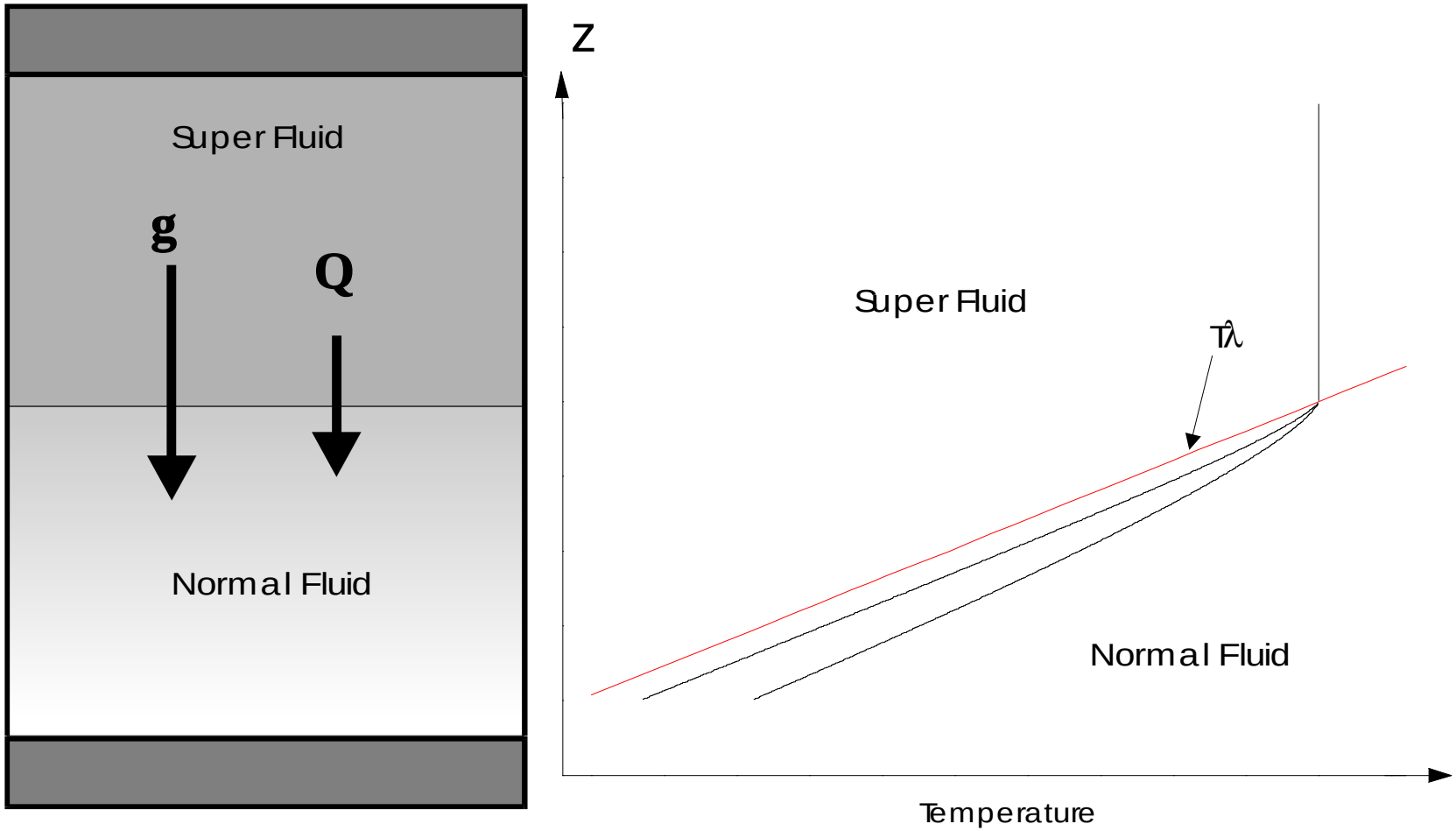
Super Fluid
g
Q
Normal Fluid
Z
Super Fluid
Tλ
Normal Fluid
Temperature

Figure 2

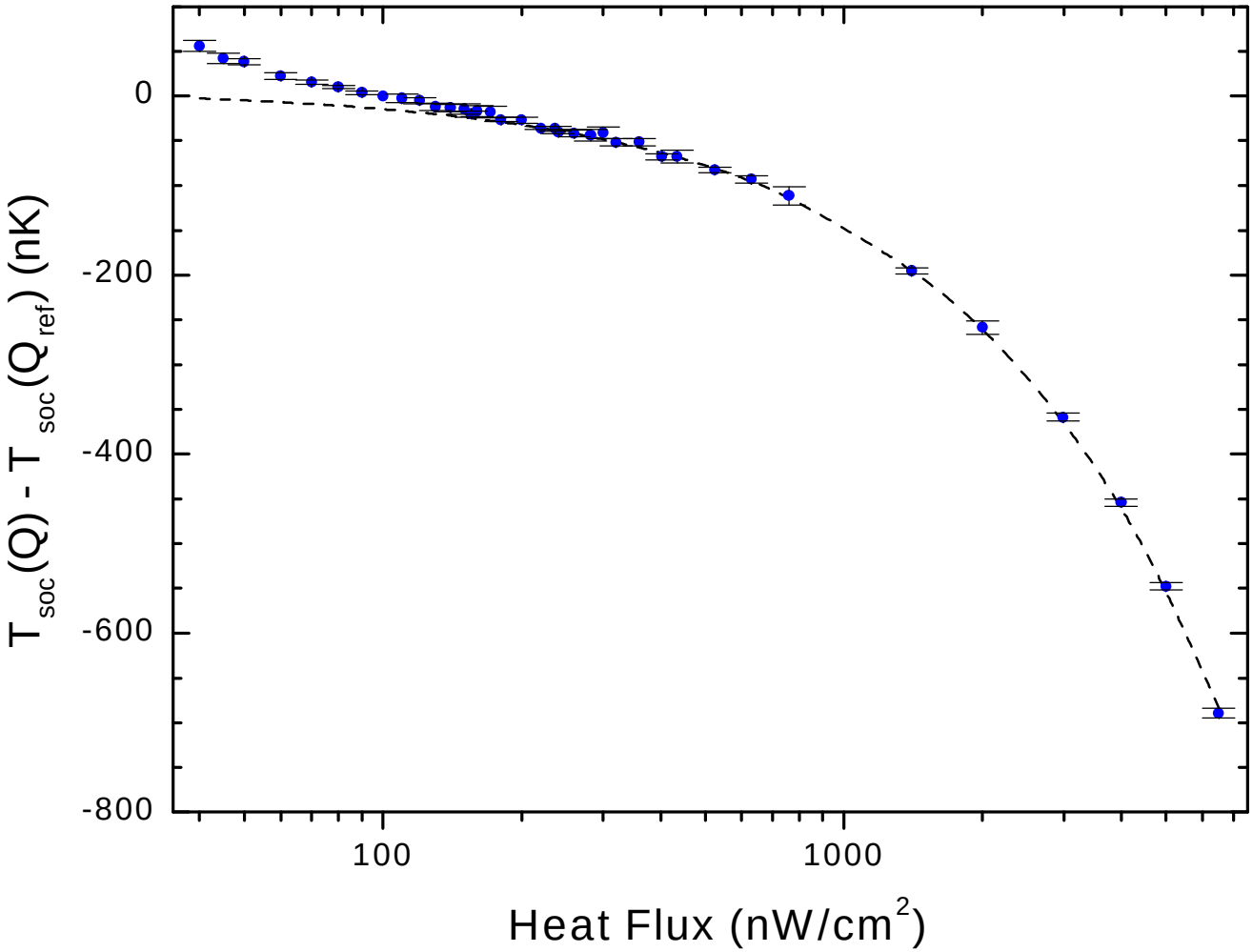

Figure 3

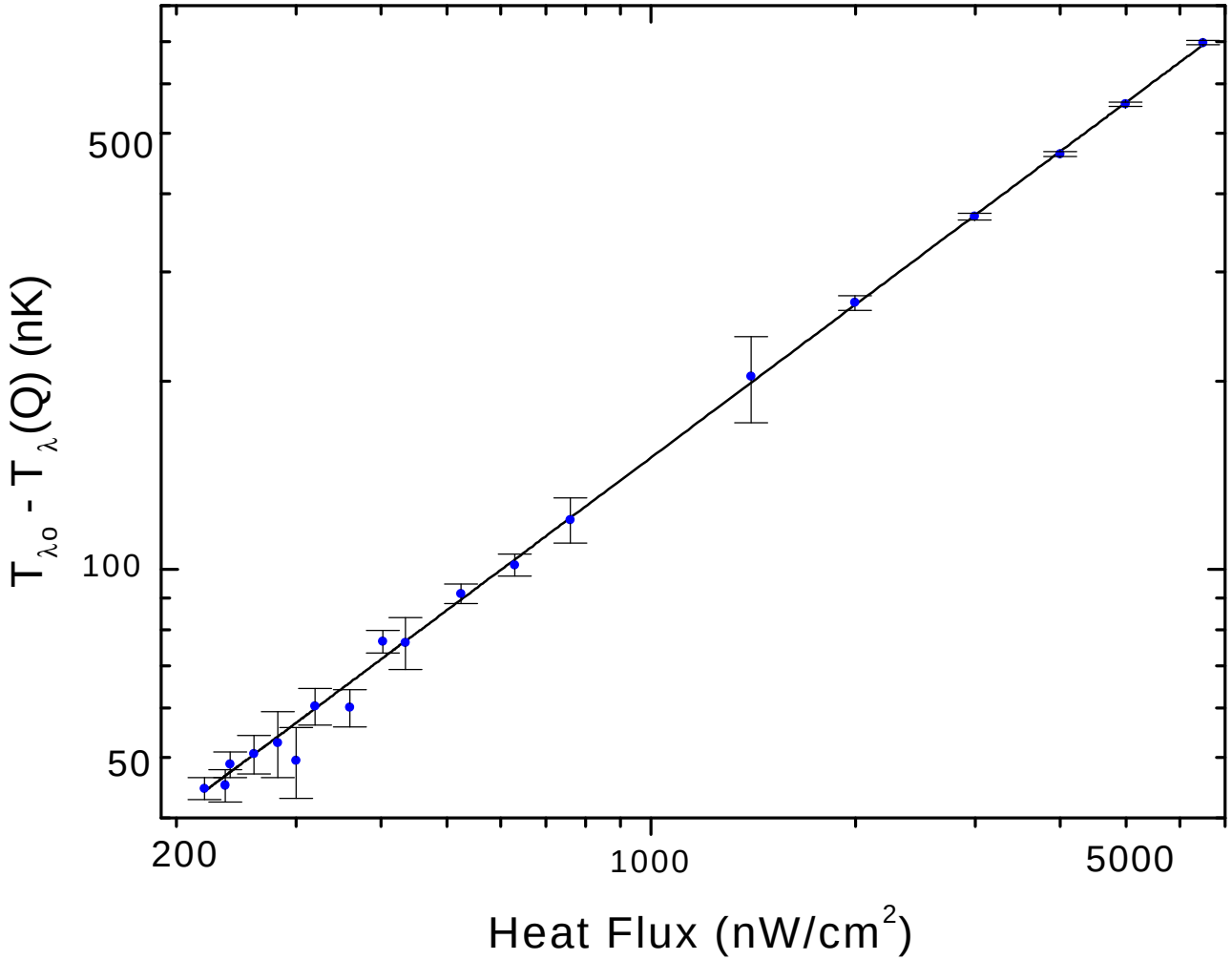
500
100
50
200
1000
5000
$T_{\lambda o} - T_{\lambda}(Q)$ (nK)
Heat Flux (nW/cm^2)

Figure 4

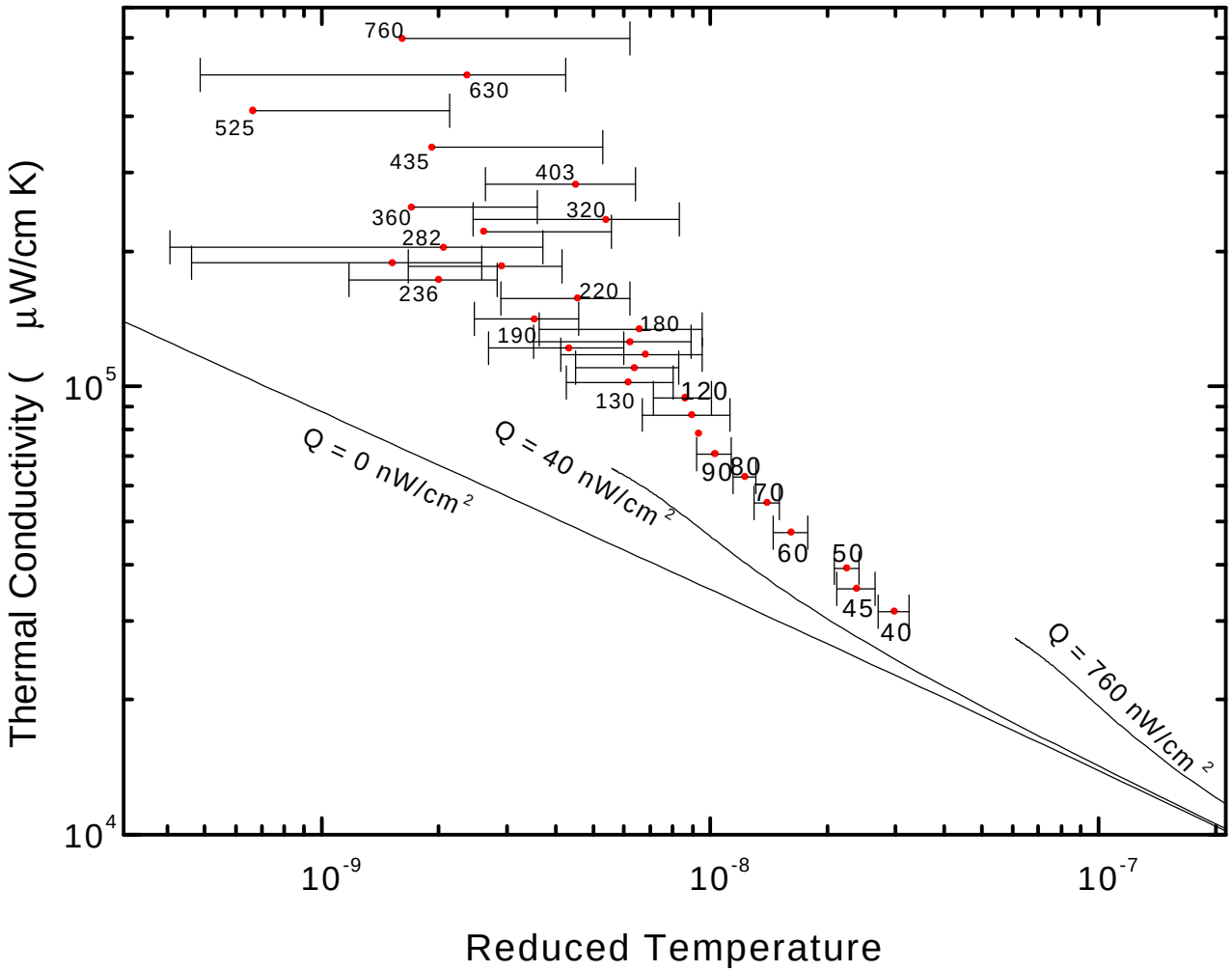
Thermal Conductivity (μW/cm K)
Reduced Temperature
10^5
10^4
10^{-9}
10^{-8}
10^{-7}
760
630
525
435
403
360
320
282
236
220
190
180
130
120
90
80
70
60
50
45
40
Q = 0 nW/cm²
Q = 40 nW/cm²
Q = 760 nW/cm²

Figure 5