Hot-Wire Measurements of the Heat Transfer in a Partially Magnetized Potassium Plasma[1]

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ABSTRACT

A hot-wire device designed as a gas-loaded heat pipe has been used to study the axial magnetic field influence on the heat transfer in a potassium vapour plasma at low vapour pressures p_K = 400 -1200 Pa, temperatures T_w of the tungsten filament in the range of 2000 to 2800 K and magnetic field intensity B = 0.182 to 0.364 T.

As a result of the applied magnetic field B the measured thermal flux Q decreases. The separation between the heat fluxes transferred by radiation Q_r, by atoms Q_a and by electrons Q_e can be accomplished analysing the decrement ΔQ. A procedure to proceed the measured data in the presence and in the absence of the magnetic field in order to define the different mechanisms of the heat transfer has been introduced. The electron thermal conductivity can be obtained in the Frost approximation by means of the well known transport phenomena theory using available electron-potassium atom cross sections in the range of electron energies e = 0.06 to 2 eV.

KEY WORDS: electron thermal conductivity; heat transfer; hot-wire measurements; partially magnetized plasma; potassium vapours.

1. INTRODUCTION

The purpose of this work is to present experimental results on the magnetic field influence on heat transport in a partially magnetized potassium plasma. The measured quantities are proceeded using different available momentum transfer electron -potassium atom cross sections in the calculations.

The interest to the transport properties of the alkali metal vapours is significant [1,2]. We have already reported results on the potassium vapour atom [3,4] and electron thermal conductivity measured by means of a hot-wire device working as a gas loaded heat-pipe. The advantages of this modification of the hot-wire method are doubtless when it is applied to the alkali metals vapours. In the traditional version of the hot-wire method the vapour pressure calculating via the measured temperature of the coldest point in the experimental device involves a large uncertainty in the obtained thermal conductivity and makes the results practically not reproducible. These problems are completely precluded in the experimental device operating as a heat-pipe: by fixing the pressure of the external inert gas (Ar) we thereby define the potassium vapours pressure.

2. MEASUREMENTS

2.1. Experimental device

The experimental device and the measurement procedure have been reported elsewhere [3,4]. In short, the gauge was a Pyrex tube with inner diameter $2R = 10^{-2}$m. A co-axial tungsten filament (diameter $2r = 1 \times 10^{-4}$ m, length 0.18 m) was heated by a stabilized *DC* current up to 2800 K and the voltage was measured with a relative error less than 10^{-5}. The system was working in a heat-pipe regime [4], the external gas being argon at a given pressure p_{Ar}. At the lower part of the gauge the potassium was evaporated. The vapours condensed in the cold part outside the device and flowed down.

With the increasing of the glass tube heating the vapours displaced the argon gas till a pure potassium atmosphere with the same pressure $p_K = p_{Ar}$ was obtained. In a gas loaded heat-pipe regime the heat losses of the wire were found to be constant in a large range of evaporation rates. The furnace containing the gauge was surrounded by a system of coaxial Helmholtz coils producing an uniform (within 1 %) axial magnetic field B up to 0.36 T.

2.2. Energy balance of the hot wire

Here we consider the tungsten wire temperatures T_w >1700 K when a plasma is generated near the filament surrounded by a gas with low ionization potential such as potassium vapours. Then the total heat flux from the filament in the hot-wire device is

$$Q = UI = Q_r(T_w) + Q_a(T_a) + Q_e(T_e) \tag{1}$$

where U and I are the filament heating DC voltage and current, respectively. The radiation (Q_r), atom (Q_a) and electron (Q_e) conductivity heat fluxes are:

$$Q_r = 2\pi rL\gamma\varepsilon_n(T_w^4 - T_R^4), \tag{2}$$

$$Q_a = A\int_{T_R}^{T_a} \lambda_a \, dT \tag{3}$$

$$Q_e = A\int_{T_R}^{T_e} (\lambda_e + \lambda_{Ri} + \lambda_{R*}) \, dT \tag{4}$$

Here $A = 2\pi L/\ln(R/r)$ is a formfactor, T_R - the cold wall temperature (~700 K); γ and ε_v are the absorption coefficient and emissivity of tungsten; λ_a and λ_e - atom and electron potassium plasma thermal conductivities. T_a and T_e stand for the temperatures of atoms and electrons near the filament, $T_a = T_w - \delta T$, δT being the temperature jump. In our experiment ionization and excitation processes can exist and this is taken into account considering the reactive thermal conductivities λ_{Ri} and λ_{R*}.

If an axial magnetic field is applied, some of electrons will stay longer in the volume between the hot wire and the cold wall. Between the collisions with heavy particles (ions and atoms) the electron moves along a trajectory determined by the cyclotron frequency $\omega = eB/m$. As a result, the perpendicular electron thermal conductivity $\lambda_{e\perp}$ and the related Q_e will decrease while both U and T_w will increase. So, we receive the basic equation of the method

$$Q(B) - Q(0) = I \times \Delta U = \frac{dQ_r}{dT_w}\Delta T_w + A\lambda_a \Delta T_a + A\left[\lambda_e \Delta T_e + \int_{T_R}^{T_e}\left(\lambda_{eB} - \lambda_e\right) dT_e\right], \qquad (5)$$

where ΔU is the wire voltage change caused by the magnetic field B. In our experiment $\Delta U/U$ and $\Delta T_w/T_w$ were in the range 10^{-3}-10^{-2} for B = 0.36 T while the term $A(\lambda_e)\Delta T_e$ is much smaller than the other terms. The change in the heat transfer due to the ionization and excitation is negligibly small compared with the other terms.

2.3. Data processing

In the experiment we measured $\Delta U = U_B - U$ at fixed p_K and B for a given set of stabilized values of I. Thus, the change of the input electrical power $\Delta Q_B = I\Delta U$ is equal to the sum of the changes of the losses caused by the wire radiation (ΔQ_r), atom (ΔQ_a) and electron (ΔQ_e) thermal conductivities

$$\Delta Q_r + \Delta Q_a - I\Delta U = \Delta Q_e \qquad (6)$$

The left side of equation (6) can be considered as experimentally determined ($\Delta Q_{exp} = I\,\Delta U - (\Delta Q_r + \Delta Q_a)$) while the right one ($\Delta Q_{calc} = \Delta Q_e$) can be calculated using the known momentum transfer cross section dependency on the electron energy $Q_{ea}(v)$. Then $y = \ln(\Delta Q_{calc}/\Delta Q_{exp})$ will be a measure of the discrepancy between the theory and experiments.

In a first approximation all the three mechanisms of the heat transfer can be considered assuming that the increases of the atom ($\Delta T_a = T_{aB}-T_a$) and electron ($\Delta T_e = T_{eB}-T_e$) temperatures are equal to the increase of the wire temperature $\Delta T_w = T_{wB}-T_w$. ΔT_w (further denoted as ΔT) can be calculated

(a) from the measured V-A characteristics as

$$\Delta T = T_{wB}(R_{01}) - T_w(R_{02}) \tag{7}$$

($R_{01} = U_B/I$ and $R_{02} = U/I$ stand for the wire resistance measured in vacuum with and without magnetic field) or

(b) using the approximation formula

$$\Delta T \approx (dT_w/dr)\,(\Delta U_B/U)\,r\,(T_W), \tag{8}$$

where $r\,(T_W)$ is tabulated tungsten resistivity [5]. The results of eq.(7) and eq.(8) coincide within 1-2%.

2.3.1.*The hot wire radiation power increment* ΔQ_r is one of the main results of the plasma magnetizing. The accuracy of its calculations defines the reliability of the final data. Three alternative approaches were used to calculate ΔQ_r:

(a) By means of the Stephan-Boltzmann low

$$DQ_r = 2pge_n\,(T_{wB}^4 - T_w^4), \tag{9}$$

where γ and ε_v are tabulated for tungsten in [5].

(b) From the V-A characteristics. We compared the points $R_1 = U_B/I$ and $R_2 = U/I$ measured in a potassium plasma at I = const with the corresponding points of the vacuum characteristics with equal $R_{10} = U_{01}/I_{01}$ and $R_2 = U_{02}/I_{02}$. Then

$$\Delta Q_r = U_{01}I_{01} - U_{02}I_{02} \tag{10}$$

(c) Using the approximation formula

$$\Delta Q_r = (dQ_r/dT_w)\,\Delta T \approx (dQ_r/dr)(\Delta U_B/U)r\,(T_w) \tag{11}$$

The derivatives (dQ_r/dT_w) and r (dQ_r/dr) were calculated for our particular wire using the tabulated tungsten temperature dependency of the specific power $q_0(T_w)$ and resistivity r (T_w) [5].

The comparison between the calculations made by means of eq.(9) - eq.(11) showed an acceptably good coincidence (≈2%). The choice of ε_v = 0.34 to 0.43 in eq.(9) resulted in a deviation of about ±5% with respect to the mean values. Therefore further on we used only eq.(9) as the simplest one.

2.3.2. *The magnetic field dependent increment of the atom conductivity heat flux*

$$\Delta Q_a = A\left\{\int_{T_R}^{T_{aB}} l_a dT - \int_{T_R}^{T_a} l_a dT\right\} \cong A l_a \Delta T \qquad (12)$$

can be calculated using our previous low temperature data on the monatomic potassium vapours thermal conductivity [3,4]

$$l_a(T) = l_a(1100\,K) \times (T_a / 1100)^{0.91} \qquad (13)$$

where λ_a(1100K)=1.62×10^{-2} W•m^{-1}•K^{-1}. The atom temperature is defined as $T_a = T_w - \delta T$. The temperature jump δT can be calculated as $\delta T = C\, Q_a T^{1/2}/p_K$ (C is a constant for a given geometry of the particular device). The calculated absolute values of ΔQ_a are approximately $0.1\Delta Q_r$.

2.3.3. *The magnetic field dependent decrement of the electron conductivity heat flux* is a result of two contradictory effects: the increase of the electron temperature ($\Delta T_e = \Delta T_w$) and the decrease of the plasma electron thermal conductivity due to the magnetizing of the electrons ($\lambda_e(B) \angle \lambda_e(0)$)

$$\Delta Q_e = A\left\{\int_{1700}^{T_{eB}} l_e(B)dT - \int_{1700}^{T_e} l_e(0)dT\right\} \cong \Delta Q_{l\,e} + A\int_{1700}^{T_e}[l_e(B) - l_e(0)]\,dT\,, \qquad (14)$$

where $\Delta Q_{\lambda e} = A\lambda_e(T_e)\Delta T$ is negligibly small compared to ΔQ_r ($\Delta Q_{\lambda e} \approx 0.01\Delta Q_r$). The calculation of the second term of Eq.(14) is based on the detailed theory of the transport properties of the partially ionized plasma developed in [6]. The Coulomb interactions are taken into account in the calculation of the electron-heavy particles collision frequency following the Frost approximation [6,7]

$$\nu^F = NvQ_T(v) + a\frac{8\pi n}{T_e^{1/2}}\left(e^2/4\pi\varepsilon_0 m\right)^2\left(m/2k\right)^2(\ln\Lambda - b/v^2)\,, \qquad (15)$$

where N is the potassium atoms density, n and m - the electron density and mass, ε_0 - electrical permittivity of vacuum, k - Boltzmann constant, $\ln\Lambda$ is Coulomb logarithm, a and b - constants different for the electrical conductivity σ, electron thermal conductivity λ_e and thermal diffusion χ [7]. $Q_T = Q_{ea}(v) + Q_{exc}(v)$ is the total electron - heavy particles interaction cross section; $Q_{exc}(v)$ being the inelastic interactions cross section.

The overheating of the plasma by the electric field ($E = U/L$) along the wire can be found from the equation

$$\sigma E^2 = \frac{3mnk}{M}\left(\bar{\nu}_{ea} + \bar{\nu}_{ei}\right)\left(T_e - T_a\right), \qquad (16)$$

where M is the potassium atomic mass, $\bar{\nu}_{ea}$ and $\bar{\nu}_{ei}$ are the mean electron-atom and electron-ion collision frequencies respectively.

The calculated increments of the heat fluxes ΔQ_r, ΔQ_a and $\Delta Q_{\lambda e}$ as functions of the wire heating current I at p_K= 800 Pa and B=0.354 T are presented in Figure 1. The corresponding relative contributions $\Delta Q_a/\Delta Q_r$ and $\Delta Q_{\lambda e}/\Delta Q_r$ of the different mechanisms in the heat transfer are shown in Table I.

3. RESULTS AND DISCUSSION

Altogether S = 82 experimental points were measured and proceeded. The experimental conditions were carefully fixed to meet the contradictory restrictions of the applied theoretical consideration. The influence of p_K, I, and B on the electron thermal conductivity in the hot-wire device is illustrated in Figures 2, 3 and 4.

In the calculation of T_e and ΔQ_e we tried different Q_{ea}(v): the semi-empirical cross section, derived from transport data [8] and the theoretical cross-sections of Spencer and Phelps [9], and Fabricant [10]. As expected, the best fit was received by a spline function minimizing only the deviations of our experiment. Unfortunately, such approximation based on an unique experiment is not universal. In the calculations of the transport coefficients we used the sum Q_{ea}(v)+Q_{exc}(v), where Q_{exc} is the total excitation cross-section to the 4P level (1.6 eV) taken from the measurements of Chen and Galagher [11].

To get an idea about the order of the values considered in this work we present in Table II an example of the calculated results at B = 0.364 T and I = 1.5 A for p_K = 400, 800 and 1200 Pa. All electron- atom cross-sections dependent values (λ_e, T_e, ΔQ_e,, $\Delta Q_{\lambda e}$, ΔQ_{calc}) were calculated with the electron velocity dependence of Q_{ea}, taken from [9]. One more justification of the choice of the experimental condition can be found in the fact that the best agreement between the theory and our experiment was obtained at p_K = 800 Pa (y = 0.087). At high pressures (p_K > 1200 Pa) the appearing breakdowns create parallel to the filament discharge channels. At low pressures (p_K <400 Pa) the accepted temperature jump approximation is not correct.

Comparing the deviations $y = \ln(\Delta Q_{calc}/\Delta Q_{exp})$ between the measured in our experiment values and those calculated using one of the available electron-atom

momentum transfer cross sections we tried to evaluate their applicability for prediction transport properties of the partially magnetized potassium plasma. In his recent work Fabricant [10] has found by a comparative analysis of alkali vapours that the theoretical momentum transfer cross sections are more reliable compared to the semi-empirical ones, particularly for reproducing swarm data. Our comparison between the standard deviation *rms* seems to support this opinion, while the mean absolute deviation $\sum y/S$ is about three times less in the case of [8]. The calculation of the electron thermal conductivity in a partially magnetized plasma is a complicated procedure. It includes calculations of the perpendicular and Hall components of the electrical conductivity and thermal diffusion. On the other hand, the estimated experimental error is relatively high (in order of 30-40 %). Because of these two inauspicious reasons we believe that both semi-empirical and theoretical cross sections could be used to interpret our experimental results.

4. **CONCLUSION**

The obtained experimental information can be used in a procedure of defining the electron-potassium atom cross section from a simultaneous fit of different type experiments. A comparison between theoretical and semi-empirical cross sections showed that both can be successfully used in numerical experiments and calculations.

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Table I. Relative Increments of the Heat Fluxes at B = 0.364 T and p_k = 800 Pa

Filament heating current (A)	1.2	1.3	1.4	1.5	1.6	1.7
$\Delta Q_a/\Delta Q_r$	0.184	0.158	0.130	0.123	0.111	0.101
$\Delta Q_{\lambda e}/\Delta Q_r$	0.0006	0.002	0.004	0.006	0.008	0.10

Table II. Potassium Vapour Pressure Influence on the Heat Transfer (B=0.364 T and I= 1.5 A, Q_{ea}(v) from [9])

p_K (Pa)	T_w (K)	T_a (K)	T_e (K)	ΔT (K)	ΔQ (W)	ΔQ_r, (W)	ΔQ_a (W)	ΔQ_{exp} (W)	ΔQ_{calc} (W)	y = $\ln(\Delta Q_{calc}/\Delta Q_{exp})$
1200	2300.5	2075.5	2352.0	2.72	0.048	0.151	0.020	0.123	0.080	0.185
800	2310.5	2001.6	2473.0	4.67	0.083	0.263	0.034	0.214	0.175	0.087
400	2315.0	1826.8	2679.0	8.91	0.158	0.506	0.060	0.408	0.516	-0.102

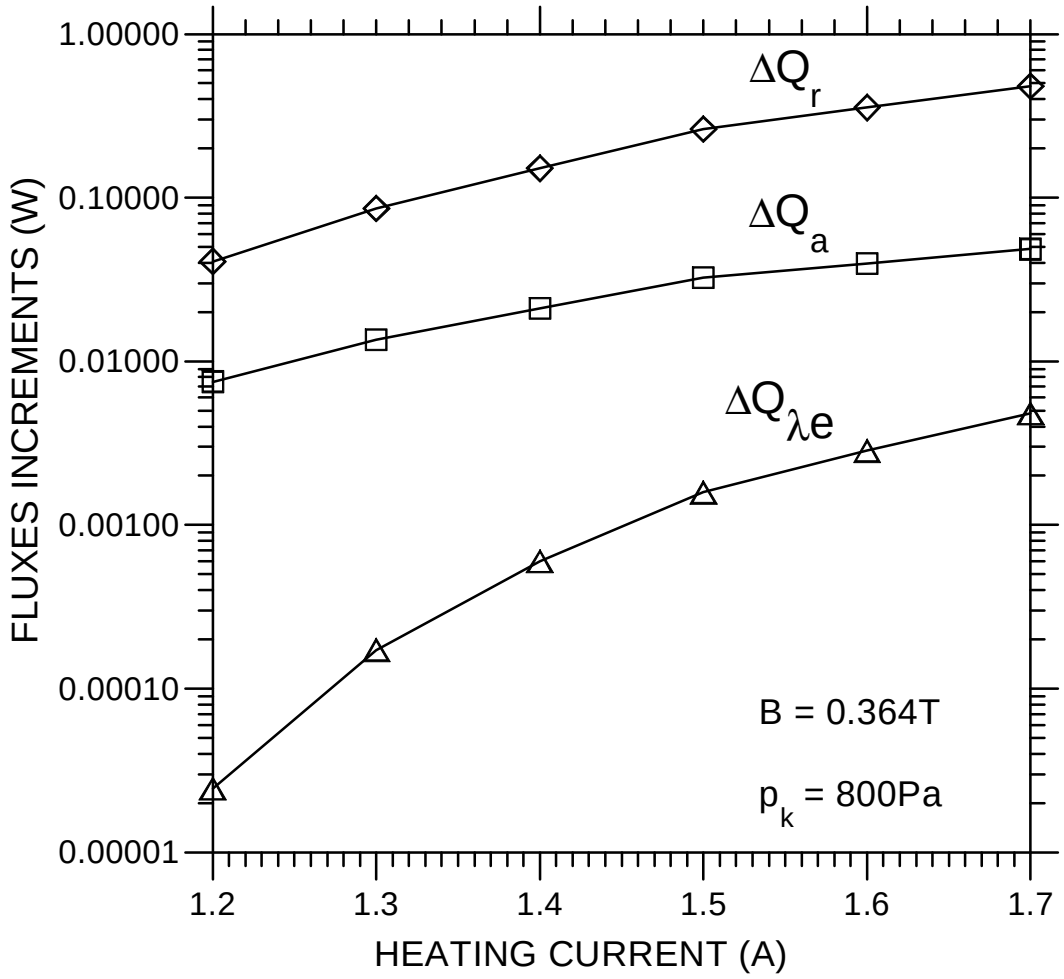

Figure 1. Increments of the heat fluxes at constant p_k and B as functions of the wire heating current: ◊- for filament radiation heat losses; □ for atom thermal conductivity heat losses and Δ- for electron thermal conductivity heat losses.

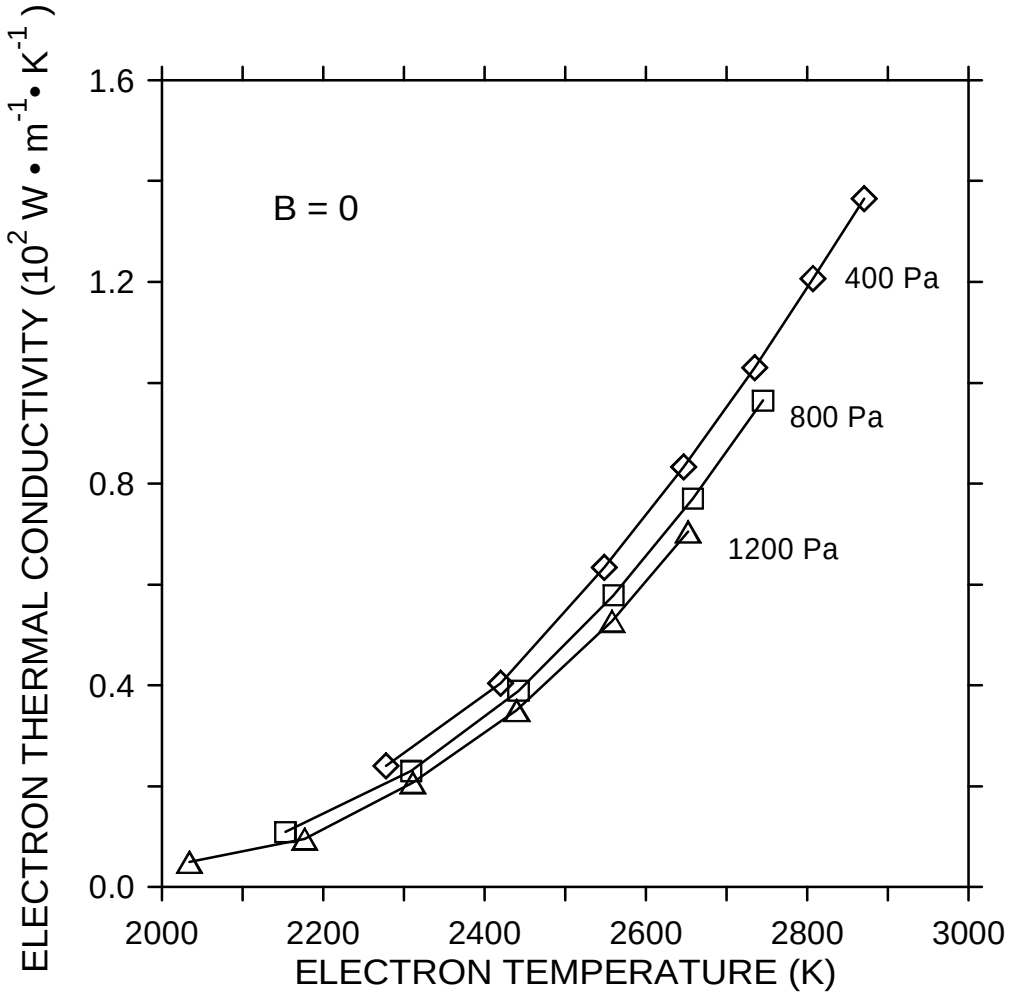

Figure 2. Temperature dependency of the electron thermal conductivity in the absence of magnetic field at different p_K: ◊ - 400 Pa; □- 800 Pa and Δ-1200 Pa.

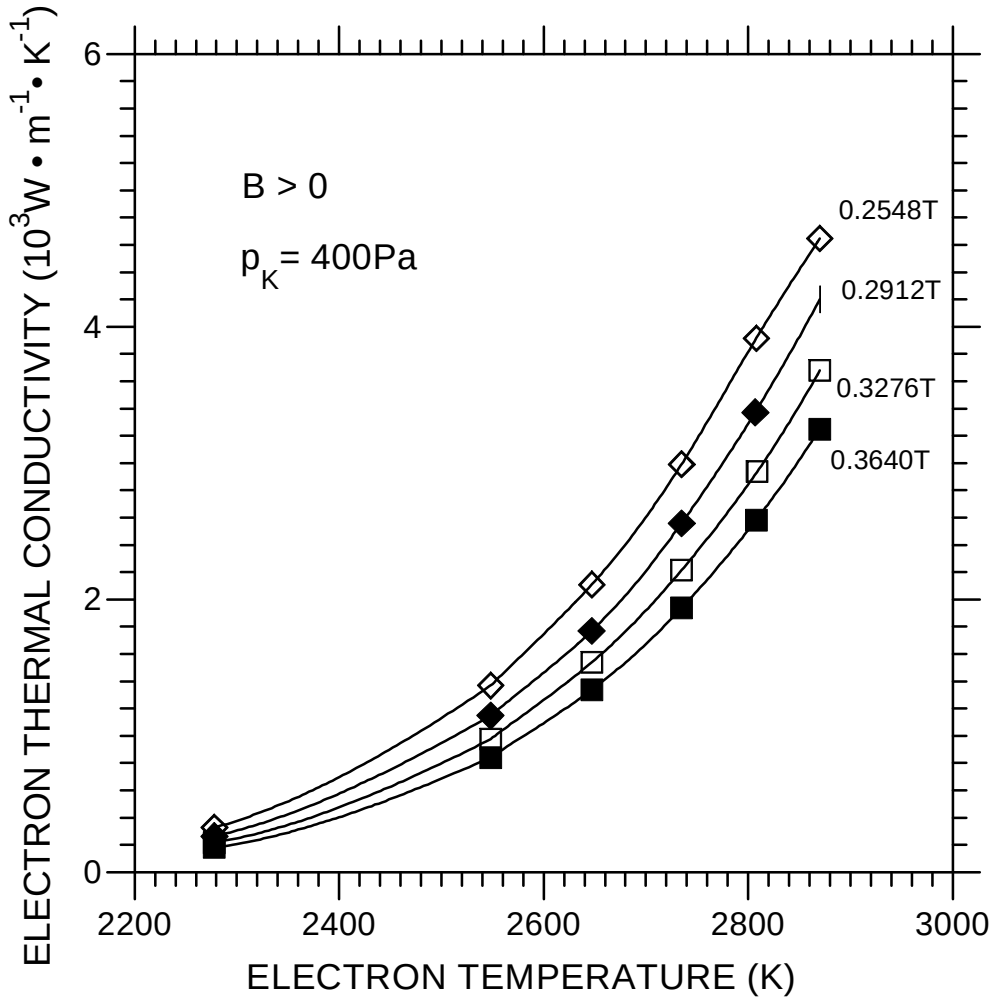

Figure 3. Temperature dependency of the electron thermal conductivity at constant p_K = 400 Pa and different magnetic field *B*: ◊ - 0.2548T; ♦- 0.2912 T; □ - 0.3276 T and - 0.3640 T.

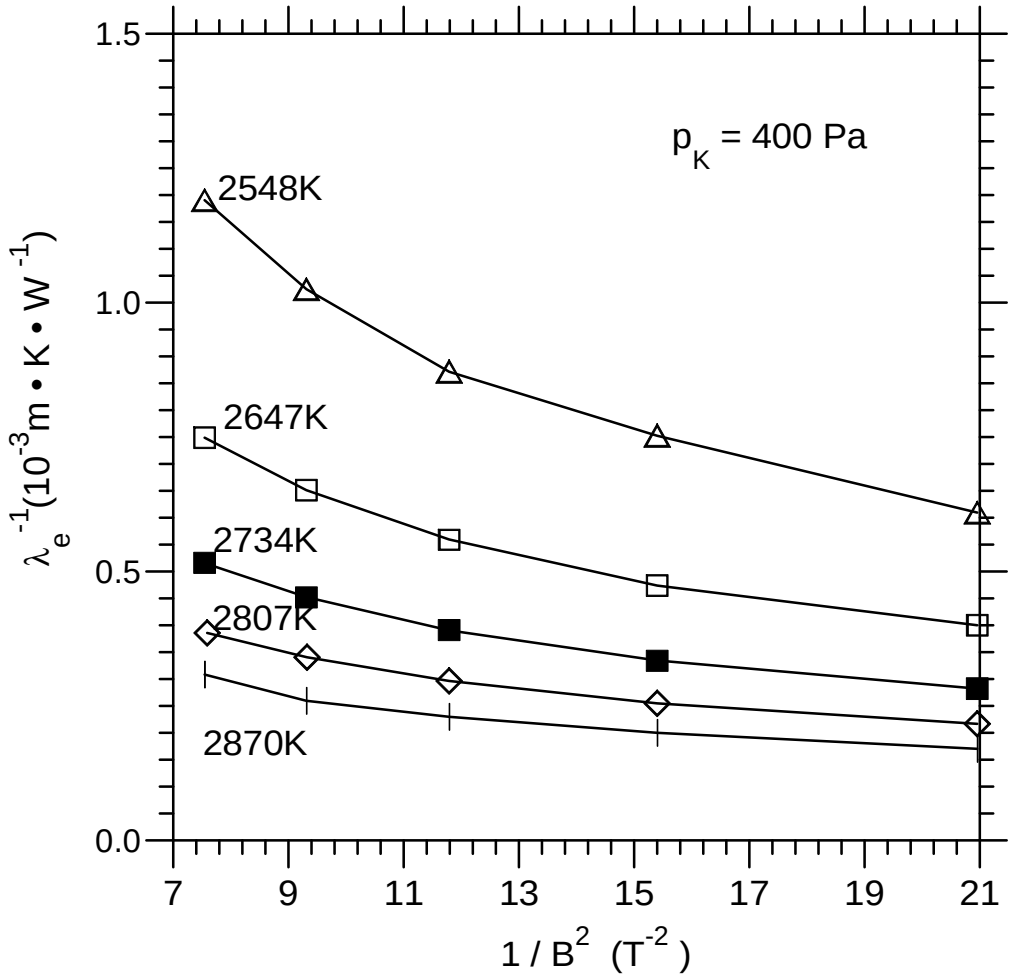

Figure 4. Electron thermal conductivity as a function of magnetic field at constant p_K = 400 Pa and different T_e: ♦- 2870 K; ◊- 2807K; -2734K; -2647K and Δ - 2548 K.