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ON NEW PHYSICAL CONSEQUENCES FOLLOWING FROM CRITICAL PHENOMENA INVESTIGATIONS. [1]

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ABSTRACT

At present the theoretical estimates of critical exponents values are obtained with high accuracy for systems with different dimensionality of space d and with different number of components N of the order parameter. For N=1 these values are approximated by equations

$$\nu = \ln 2 / \ln d, \quad \eta = [(4-d)/(2+d)]^2 \tag{1}$$

where ν and η are the critical exponents of correlation radius and correlation function. Attemps to use the values of critical exponents of three - dimensional model systems for the description of liquid - gas system experimental data lead to a very narrow region of action of asymptotic equations and to some distortion of common regularities of the thermodynamic properties behavior. New experimental data analysis in accordance with authors' method leads to necessity to use the so called "compromise" (experimental) critical exponents. These exponents extend region of action of asymptotic equations and are in good accordance with common regularities of the thermodinamic properties behavior. The elimination of discrepancy between theoretical and experimental values of critical exponents is possible when noninteger dimensionality of space $d \approx 3.1$ is used in equations (1). There are two reasons to explain the above mentioned:

1) Critical exponents depend on additional to d and N third parameter X which is the feature of the liquid - gas system;

2) Critical exponents depend only on d and N. Physical space has noninteger dimensionality which is the feature of the fractal geometric objects.

KEY WORDS: critical exponent, critical parameters, dimensity of space, fractal object, individual substances, order parameter number, thermodynamic properties.

1. INTRODUCTION.

Thermodynamic values behavior of different physical systems near its critical points is determined by simple power laws of the kind

$$\Delta y = E\ |\Delta x|^{\varepsilon} \tag{1}$$

where $\Delta y = (y-y_c) / y_c$, $\Delta x = (x-x_c) / x_c$, "c" is the sign of critical state, ε - critical exponent, E - cirtical amplitude. Existing theoretical estimates of critical exponents values one can describe by different analytical dependences on space dimensity d of physical system [1, 2].

At first the question "In what way does it become manifest in the fundamental laws of physics that space has three dimensions?" was formulated by P. Ehrenfest in 1917 [3, 4]. He had investigated sencibility of planet system, Bohr's spectrum of hydrogenium atom and wave process to the value of space dimensity $d = 2$, $d = 3$ and $d \geq 4$. On Ehrenfest's view the space dimesionality penetrates all the physics and thus becomes the physical concept.

Critical exponents are directly related by sensible dependence to space dimensionality. Such relation gives new trend for critical phenomena investigations and space dimensionality becomes the physical value which one can determine from accurate experimental data on thermodynamic properties of substances.

Our new analysis of experimental data on thermodynamic properties of water [5-7] shows that critical exponents of liquid - gas system are nearer to the previous "compromise" estimates [8, 9] than to the theoretical estimates for three - dimensional systems $d = 3$ with one - component order parameter $N = 1$. At the same time the substitution of noninteger space dimensionality $d = 3.1$ into theoretical formulas gives the experimental values of critical exponents for the liquid - gas system. There are two reasons to explain the above mentioned:

1) Critical exponents depend on additional to d and N third parameter X which is the feature of the liquid - gas system;

2) Critical exponents depend only on d and N. Physical space has noninteger dimensionality which is the feature of the fractal geometric objects.

In this paper dependences of critical exponents on d and N are presented, arguments are adduced for declination of experimental critical exponents from

theoretical one for d = 3 and N = 1, some features of fractal space are considered and the possibilities of such mathematical model of space using for physical phenomena description are discussed.

2. THE DEPENDENCE OF CRITICAL EXPONENTS ON THE DIMENSIONALITY OF SPACE AND THE NUMBER OF THE ORDER PARAMETER.

In [1] we proposed the formulas for critical exponents of correlation radius ν and correlation function η

$$\nu = \frac{2}{d} + \frac{(d-2)(d-4)(2-N)}{d^3} \quad (2), \qquad \eta = \frac{4-d}{d^3 + d^2 - 4} \quad (3)$$

which are consistent with the theoretical estimates of critical exponents obtained by 1984 for d = 2, 3, 4 and N = 1, 2, 3. In [2] we proposed the formulas

$$\nu = \frac{\ln 2}{\ln d} \quad (4), \qquad \eta = \left(\frac{4-d}{2+d}\right)^2 \quad (5)$$

which are consistent with the new theoretical estimates of critical exponents in the region d = 1 - 4 at N = 1. On Fig.1 and Fig.2 the dependences $\nu(d)$ (4) and $\eta(d)$ (5) are compared with critical exponents estimates [10]. Other critical exponents are connected with critical exponents ν and η by scaling theory relations

$$\alpha = 2 - d\nu \quad (6), \qquad \beta = \frac{\nu}{2}(d - 2 + \eta) \quad (7)$$

$$\gamma = \nu(2 - \eta) \quad (8), \qquad \delta = \frac{d + 2 - \eta}{d - 2 + \eta}. \quad (9)$$

Substituting equations (4) and (5) into relations (6) - (9), we express the remaining critical exponents of systems with a one - component order parameter in terms of the dimensionality of space:

$$\alpha = 2 - d \ln 2/\ln d \quad , \tag{10}$$

$$\beta = \frac{\ln 2}{2 \ln d}\left[d - 2 + \left(\frac{4 - d}{2 + d}\right)^2\right] \quad , \tag{11}$$

$$\gamma = \frac{\ln 2}{\ln d}\left[2 - \left(\frac{4 - d}{2 + d}\right)^2\right] , \tag{12}$$

$$\delta = \left[d + 2 - \left(\frac{4 - d}{2 + d}\right)^2\right] \Big/ \left[d - 2 + \left(\frac{4 - d}{2 + d}\right)^2\right]. \tag{13}$$

By eliminating d from the equations for two exponents, one critical exponent may be expressed in terms of another. For example, equations (4) and (5) may be written in the form

$$d = 2^{1/\nu} \qquad (14), \qquad d = 2\,(2 - \eta^{1/2})/(1 + \eta^{1/2}) \ , \tag{15}$$

$$\nu = \frac{\ln 2}{\ln 2 + \ln (2 - \eta^{1/2}) - \ln (1 + \eta^{1/2})} \quad . \tag{16}$$

For the d = 3.0 - 3.3 range the critical exponents α, γ, δ, calculated by equations (10) - (13), may be represented as the following functions of the critical exponent β:

$$\alpha = -0.01692 + 1.0679\,\beta - 2.1016\,\beta^2 \quad , \tag{17}$$

$$\gamma = 2.017 - 3.0684\beta + 2.1023\beta^2 \quad , \tag{18}$$

$$\delta = 15.0583 - 46.308\,\beta + 45.55\,\beta^2 \quad . \tag{19}$$

Simple approximating formulas can also be constructed for the dependence of critical exponents on the number of components of the order parameter. The

existing theoretical estimates of critical exponents values for d = 2, 3 and N = 0, 1, 2, 3 may be reproduced by formula (5) for η and by the next formula for ν

$$\nu = \frac{\ln 2}{\ln d} + (N - 1)\left(\frac{4 - d}{2 + d}\right)^2. \qquad (20)$$

To describe the critical exponents of the spherical model corresponding to d = 3 and N = ∞, formulas for η (d, N) and ν (d, N) must yield $\eta = 0$ and $\nu = 1$ as $N \to \infty$. We propose the following modification of formulas (4) and (5), which ensures the correct transition to the values of critical exponents of the spherical model:

$$\eta = \left(\frac{4 - d}{2 + d}\right)^2 \frac{2N - 1 + d^3}{N^2 + d^3} \quad (21), \qquad \nu = \ln 2/\ln\left(d\,\frac{2N + 1 + d^2}{3N + d^2}\right). \quad (22)$$

Fig.3 shows dependence β on δ at fixed value N = 1 and at arbitrary values d (curve 1). On this curve point 7 marks theoretical critical exponents corresponding to d=3. Experimental estimates of the critical exponents values of the liquid-gas system [8] (point 6), [7] (point 5) and [9] (point 4) correspond to space dimensionality d = 3.06, 3.10, 3.15 respectively. Passing through point 7 lines 2 and 3 are calculated by the use of formulas (5), (20) and (21), (22) and relations (7), (8) at fixed value d = 3 and at arbitrary values N. In this case the critical exponent β is independed on the critical exponent δ. Thus the declination of theoretical values of critical exponents $\beta = 0.328 \pm 0.003$ and $\delta = 4.77 \pm 0.05$ from experimental estimates for liquid-gas system $\beta = 0.35 \pm 0.01$ and $\delta = 4.43 \pm 0.15$ may be eliminated only by the change of space dimensionality from 3 to ~3.1 (transition from point 7 into point 5).

3. THE EXPERIMENTAL ESTIMATES OF CRITICAL EXPONENTS VALUES FOR THE LIQUID-GAS SYSTEM.

The purely power dependence of the form of equation (1) is almost never realized for the thermodynamic functions of real substances, and one usually observes the more complex case

$$\Delta y = E|\Delta x|^{\varepsilon}\,[1 + f(\Delta x)] \quad . \tag{23}$$

That is why the experimental estimates ε^{ef} away from the critical point will differ from the true value of the critical exponent ε

$$\varepsilon^{ef} = \frac{d\ln|\Delta y|}{d\ln|\Delta x|} = \varepsilon + \frac{\Delta x\, df(\Delta x)/d\Delta x}{1 + f(\Delta x)} \quad . \tag{24}$$

Therefore, in order to estimate the critical exponent reliably, one needs to elimimate or at least weaken the distorting effect of correction terms to the asymptotic dependence and of indeterminacy of the choice of values of the critical parameters, that is, find a method to expand the range of validity of the purely power dependence (1). One such method was used in [5] to determine the critical exponent β from the experimental data on density of water on the boundary curve. The variable Δx in equation (23) is transformed so that the dependence of the quantity Δy being investigated on the new variable

$$\Delta x^* = \Delta x\,[1 + \varphi(\Delta x)] \tag{25}$$

would be purely power dependence

$$\Delta y = E\,|\Delta x^*|^{\varepsilon} \quad . \tag{26}$$

Another possible method of determining the critical exponent ε is the direct extrapolation of the relationship $\varepsilon^{ef}(\Delta x)$, derived from experimental data, to $\Delta x = 0$.

Dependence of density on temperature on the boundary curve may be presented in the form

$$\Delta\varrho = \pm\Delta\varrho_1 + \Delta\varrho_2\,,\quad \Delta\varrho_1 = (\varrho' - \varrho'')\,/\,2\varrho_c,\ \Delta\varrho_2 = (\varrho' + \varrho'')\,/\,2\varrho_c - 1 \qquad (27)$$

where the indexes "′" and "″" denote the parameters of liquid and gas state. Dependence of density on pressure on the critical isotherm may be pesented in the form (27) as well by turning the gas branch of the critical isotherm toward high pressures. In this case the values of $\varrho'$and $\varrho''$are taken for the same values $|\Delta P| = |P - P_c|/P_c$. Asymptotic benavior of $\Delta\varrho_1$ on the boundary curve and on the critical isotherm is determined by power laws

$$\Delta\varrho_1 = B|t|^{\beta} \qquad (28), \qquad\qquad \Delta\varrho_1 = D^{-1/\delta}\,|\Delta P|^{1/\delta}\ . \qquad (29)$$

The use of dependences (28) and (29) for determination of the critical exponents β and δ eliminates the distorting effect of the asymmetric part of density and indeterminacy of the choice of value of the critical density.

However, since the choice of the form of transformation $\Delta x \to \Delta x^*$ or of extrapolation of $\varepsilon^{ef}(\Delta x)$ to $\Delta x = 0$ is largely arbitrary, the reliability of these and other methods of determining critical exponents from the results of analysis of experimental data needs to be checked in model systems for which the exact values of critical exponents are known.

As model system we will use Van-der-Waals` equation of state in the form [11]

$$\left(\Delta P - \frac{K}{1-b}\,t\right)\left(1 - \frac{b}{1-b}\,\Delta\varrho\right) = \frac{1}{1-b}\,\Delta\varrho^3 + \frac{K}{(1-b)^2}\,t\Delta\varrho \qquad (30)$$

and the equation of state of the form [12]

$$\left(\Delta P - \frac{K}{1-b}\,t\right)\left(1 - \frac{b}{1-b}\,\Delta\varrho\right) = \frac{1}{1-b}\,\Delta\varrho^5 + \frac{K}{(1-b)^2}\,t\,\Delta\varrho\ . \qquad (31)$$

From equations (30) and (31), one can calculate the values of any thermodynamic function when apporoaching the critical point from any direction, with any preset accuracy. Therefore, equations (30) and (31) may be used to generate model sets of experimental data of preset accuracy, which must lead to two different sets of values of critical exponents, namely, $\beta = 0.5$ and $\delta = 3$, and $\beta = 0.25$ and $\delta = 5$.

These values of β and δ limit from above and below all known experimental estimates of critical exponents of individual substances.

Figure 4 gives the $\beta^{ef}(t)$ relationships, calculated from the values of $\Delta\varrho_1$ by the tabular data [13] for water, as well as from the values of $\Delta\varrho_1$ calculated by equations of state (30) and (31), and from the equation for spontaneous magnetiration of diamond lattice of Ising model [14]

$$M = B\ |t|^{\beta} / [1 + b_1\ |t|^{\Delta_1} + b_2\ |t|^{\Delta_2}] \qquad (32)$$

where $\beta = 0.3225$, $\Delta_1 = 0.5$, $\Delta_2 = 1$, $B = 1.6$, $b_1 = 0.168$, $b_2 = 0.432$.

The values of β^{ef} for both water and model systems increase as the critical point is approached. The estimates of the critical exponent β for water and model equations of state for different methods of approximation the relationship $\beta^{ef}(t)$ in the range $(-t) = 0.01 - 0.11$ are given in Table 1. In case of appropriate choice of the form of approximation dependence, the error of determination β for model equations of state is less than 0.1%. For water, the values of the critical exponent derived by all approximation methods, lie in the narrow range of $\beta = 0.34666$ - - 0.34722 and agree well with the estimate of $\beta = 0.34613$ [5] derived earlier as a result of approximation of the same array of initial values of $\Delta\varrho_1$ by the equation

$$\Delta\varrho_1 = B\ |t\ (1 - 0.7665|t|)|^{\beta}\ .$$

The similar results were obtained for other substances [7].

These regularities in the behavior of β^{ef} suggest that the value of the critical exponent β of individual substances is higher than the value $\beta = 0.328$ predicted by the lattice gas model, because the relationship $\beta^{ef}(t)$ is close to linear and passes through $\beta = 0.328$ at a cosiderable distance from the critical point (for water, at $(-t) \sim 0.06$). The probable value of the critical exponent for water is $\beta = 0.346$ - 0.348.

Figure 5 gives the relationships $\delta^{ef} = f(|\Delta P|^{1/\delta})$, calculated by equations of state (30) and (31), as well as the values of δ^{ef} calculated by using the values of $\Delta\varrho_1$ isolated from experimental P, ϱ, T - data for water [15] and argon [16] on the critical isotherm. For model equations of state, the values of δ^{ef} vary but little with increasing distance from the critical point, while they decrease (equation

(30)) and increase (equation (31)) with increasing $|\Delta P|$. For real substances, the values of δ^{ef} decrease with increasing $|\Delta P|$, while the probable values of the critical exponent $\delta = 4.41 - 4.49$ correspond to the space dimension $d \sim 3.1$ in equation (13) and agree with independent estimates of the critical exponent $\beta = 0.346 - 0.351$.

4. THE FEATURES OF FRACTAL SPACE.

The noninteger space dimensionality is the feature of the fractal objects. There are different definitions of fractals because of a great number of its properties [17- - 19]. We shall consider such fractal objects that correspond to the definition: the finite volume $V^F{}_d$ in the space of the d dimensions, which is restricted by the infinite surface $S^F{}_{d-1}$ in the space of the (d - 1) dimensions. The dimensionality of the surface of such fractal object has noniteger value $(d - 1) < d_F < d$ and is determined by the equation [18]

$$d_F = \lim_{m \to \infty} \frac{\ln N(y, m)}{\ln y^m} \tag{33}$$

where N (y, m) is the minimum number of hypercubes of dimensionality (d - 1) with side y^{-m} necessary for covering the surface of the vintial d - dimensional object. An example of a fractal object is providend by Koch's line thet is constructed as follows (Figure 6). Every side of an equilateral triangle ABC is divided into three equal parts, and a new equilateral triangle is constructed on the outside of every middle part of sides of triangle. By iterating this process ad infinitum, we derive an infinite line $S^F{}_1$ restricting a finite area $V^F{}_2$. On the analogy, fractal surface $S^F{}_2$ and fractal space $S^F{}_3$ may be constructed on the basis of a tetrahedron or cube and four-dimensional hypertetrahedron or hypercube.

At m-fold decreasing of triangle`s side into $y^m$ times $N(y, m) = 3 \cdot 4^m$ and dimensionality of Koch`s line is

$$d_F = \lim_{m \to \infty} \frac{\ln (3 \cdot 4^m)}{\ln 3^m} = \lim_{m \to \infty} \left(\frac{\ln 3}{m \ln 3} + \frac{m \ln 4}{m \ln 3} \right) = \frac{\ln 4}{\ln 3} = 1.26186.$$

In the common case the dimensionality of the fractal surface S^F_{d-1} is determined by the equation

$$d_F = \frac{\ln P}{\ln y} = \frac{\ln [y^{d-1} + n (q-2)]}{\ln y} \tag{34}$$

where y is the coefficient of reduction of the side of the initial object; n is the number of reduced objects extended from one face to the outside; d is the dimension of the initial object; and q is the number of (d-1) - dimensional faces of the d - dimensional initial object.

The area restricted by Koch's line is

$$V^F_2 = V_2 + 3\frac{V_2}{9}\left[1 + \frac{4}{9} + \ldots + \left(\frac{4}{9}\right)^m + \ldots\right] = V_2\left(1 + \frac{3}{9}\cdot\frac{1}{1-4/9}\right) = 1.6\ V_2$$

where V_2 is area of initial triangle ABC.

In the common case the value of volume of fractal object V^F_d with initial value of volume V_d is determined by the equation

$$V^F_d = V_d\left(1 + \frac{qn}{y^d - p}\right). \tag{35}$$

Thus for the fractal object V^F_d one can introduce the absolute length

$$L = \left[V_d\left(1 + \frac{qn}{y^d - p}\right)\right]^{1/d}. \tag{36}$$

On the other hand the infinite set of partial lengths $l_m \sim y^{-m}$ is needed for the description of the fractal surface S^F_{d-1}. Any preset scale of length l_m is related with indeterminacy in value of the fractal object volume. There is lost of information on the area of the fractal object V^F_2 given by Figure 6 1/5 V_2, 4/45 V_2 and 16/405 V_2 when one moves from point A to point B, from point B to

point C and from point C to point A respectively. But this indeterminacy has the universal value $\chi = 1/5$ when it is related to the summar area of thiangles with the length l_m. In the common case the constant χ is determined by the equation

$$\chi = \frac{V^F_d - V^F_d\,(m)}{N(y,\,m)\,V_d(y,m)} = \frac{n}{y^d - p} = \frac{n}{y^{d-1}(y-1) - n(q-2)} = \frac{n}{y^d - y^d_F} \qquad (37)$$

where V^F_d (m) is the value of volume of subfractal at m-step iteration and V_d (y,m) is the value of elementary volyme at m-step iterasion.

The character properties of some fractal spaces S^F_3 which are constructed on the basis of the four-dimensional hypertetrahedron and the four-dimensional hypercube are presented in the Table 2.

5. THE POSSBLE CONSEQUENCES OF THE DECLINATION OF EXPERIMENTAL VALUES OF THE LIQUID-GAS SYSTEM CRITICAL EXPONENTS FROM THE THEORETICAL ONE FOR THE THREE-DIMENSIONAL SYSTEM WITH ONE-COMPONENT ORDER PARAMETER.

The first experimentally based attempt to revise the classical values of the critical exponents was done by Guggenheim in 1945, when he showed that the critical exponent $\beta = 1/3$ gives more accurate description of the boundary of real fluids than the classical critical exponent $\beta = 1/2$ [20]. But the final revision of the classical values of critical exponents was done only in twenty years, after the possibility of existence of nonclassical critical exponents had been theoretically grounded by estimates for the three-dimensional Ising model and the scaling equation of state had been developed for description of the thermodynamic surface of individual substances by use of the nonclassical critical exponents. The experimental "compromise" critical exponent β which was some more 1/3, and the theoretical critical exponent β, which was some less 1/3, "coexisted" some time. It was supposed that more accurate estimates of experimental and theoretical values of critical exponents would coincide. But by the 1980 it became finally clear

that theoretical values are in the limits $\beta = 0.32 - 0.33$, while for description of experimental data it is preferable to use the values $\beta = 0.34 - 0.36$. Thus as the result of progress of the modern theory of critical phenomena the problem of determination of true values of critical exponents of liquid-gas system was transformed into problem of approximation of experimental data so that for their description one could use theoretical values of critical exponents.

But at more careful analysis the modern theory of critical phenomena gives, at least, some formal foundation for existence of the "compromise" critical exponents. Critical exponents are directly related by sensitive dependence to the space dimension. "Compromise" exponents one can obtain, when space dimensionality d=3.1 is substituted into the theoretical formulas. More careful analysis of experimental data and investigation of dependences of effective exponents on parameters of state for individual substances and model systems give foundation to say that true values of critical exponents of liquid-gas system are in the limits $\beta=0.35 \pm 0.01$ and $\delta=4.43 \pm 0.15$ while theoretical values for three-dimensional systems with one - component order parameter are $\beta=0.328 \pm 0.003$ and $\delta=4.77 \pm 0.05$.

One of possible explanations of this discrepancy is the existence of additional to d and N third parameter X which is the feature of the liquid-gas system. Thus theoretical dependences for critical exponents must use (d+X) instead of d with $X \approx 0.1$ for liquid-gas system.

Another possible explanation of declination of experimental critical exponents from theoretical one is that value $d \approx 3.1$ is the true dimensity of physical space. In this case one can use the four-dimensional fractal object $V^F{}_4$ as mathematical model of space-time continuum and the fractal surface $S^F{}_3$ of this object is mathematical model of physical space.

In the common theory of relativity the mathematical model of physical space is the three-dimensional surface of some four-dimensional geometric object as well. But in the case of fractal space $S^F{}_3$ this surface is not "smooth" but "rough". As the result of such "roughness" the space is infinite and has the noninteger dimensity $3 < d_F < 4$, but at any chosen scale of length l_m the space is finite and three-dimensional. Any preset scale of length is connected with relative space - time indeterminacy χ, which is the universal constant of space.

In the modern phylosophical literature the problems of quant properties of space and time, characteristic scale of length, hierarchy of times are discussed. Such properties are the features of the fractal model of space - time continuum. That is why the hypothesis on fractal character of physical space is rightful and needs serious mathematical, physical and phylosophical studing.

It is possible that future analysis may establish the quantitative relation of different physical phenomena with the dimensity of the space. For the critical phenomena such dependence has already been established: critical exponents, which deptermine asymptotic behavior of thermodynamic functions in different physical systems are directly related by the sensitive dependences to the space dimension. Thus the space dimensity becomes not only the physical concept, but the quantitative measure which one can determine through immediate measurements of physical properties of substances.

6. CONCLUSION.

Results presented in this paper give foundation to say that critical exponents of liquid-gas system differ from theoretical prediction for the three-dimensional systems with one-component order parameter. But this difference may become the initial point for the new stage of critical phenomena investigations. As there exists the direct relation of critical exponents to the space dimensity, the final solution of the particular problem of description of thermodynamic functions behavior in the critical region may answer the most fundamental question - what is the World we are living?

That is why we can forecast a new rise of critical phenomena investigations as one of significant instruments for establishment of fundamental regularities of nature.

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Table 1. Estimates of values of the critical exponent β for water and for equations of state (30) and (31) using different methods of the relationship β^{ef} (t) approximation.

Approximation dependence	Water	Equation (30)		Equation (31)	
	β	β	$\delta\beta$, %	β	$\delta\beta$, %
$\beta^{ef} = \beta + a_1\|t\|$	0.34722	0.50054	0.108	0.24319	-2.72
$\beta^{ef} = \beta + a_1\|t\| + a_2\|t\|^2$	0.34704	0.50018	0.036	0.24503	-1.99
$\beta^{ef} = \beta + a_{1/2}\|t\|^{1/2} + a_1\|t\|$	0.34666	0.49936	-0.128	0.24987	-0.052

Table 2. Characteristics of fractal spaces $S^F{}_3$ made on the basis of the four - dimensional hypertetrahedron and hypercube.

y	q	n	p	d_f	χ^{-1}	α	β	γ	β	ν	η
3	5	1	30	3.0959	51	0.1011	0.3457	1.2074	4.4922	0.6134	0.0315
3	8	1	33	3.18266	48	0.0945	0.3615	1.1825	4.2714	0.5987	0.0249
4	8	1	70	3.06464	186	0.1032	0.3400	1.2167	4.5784	0.6189	0.0341
4	8	2	76	3.12396	90	0.0991	0.3509	1.1992	4.4179	0.6085	0.0292
7	8	12	415	3.09792	165.5	0.1009	0.3461	1.2068	4.4868	0.6130	0.0313

Figure Captions

Fig.1. Comparison of the correlation radius critical exponent ν calculated by formula (4) with data [10].

Fig.2. Comparison of the correlation function critical exponent η calculated by formula (5) with data [10].

Fig.3. The dependence of the critical exponent β on the critical exponent δ.

Fig.4. The dependence of the effective exponent of boundary β^{ef} on temperature: (1) Van der Waals' equation (30), (2) water, (3) equation (32), (4) equation of state (31).

Fig.5. The dependence of the effective exponent of critical isotherm δ^{ef} on pressure: (1) equation of state (31), (2) water, (3) argon, (4) van der Waals' equation (30).

Fig.6. The scheme of Koch's line constructing.

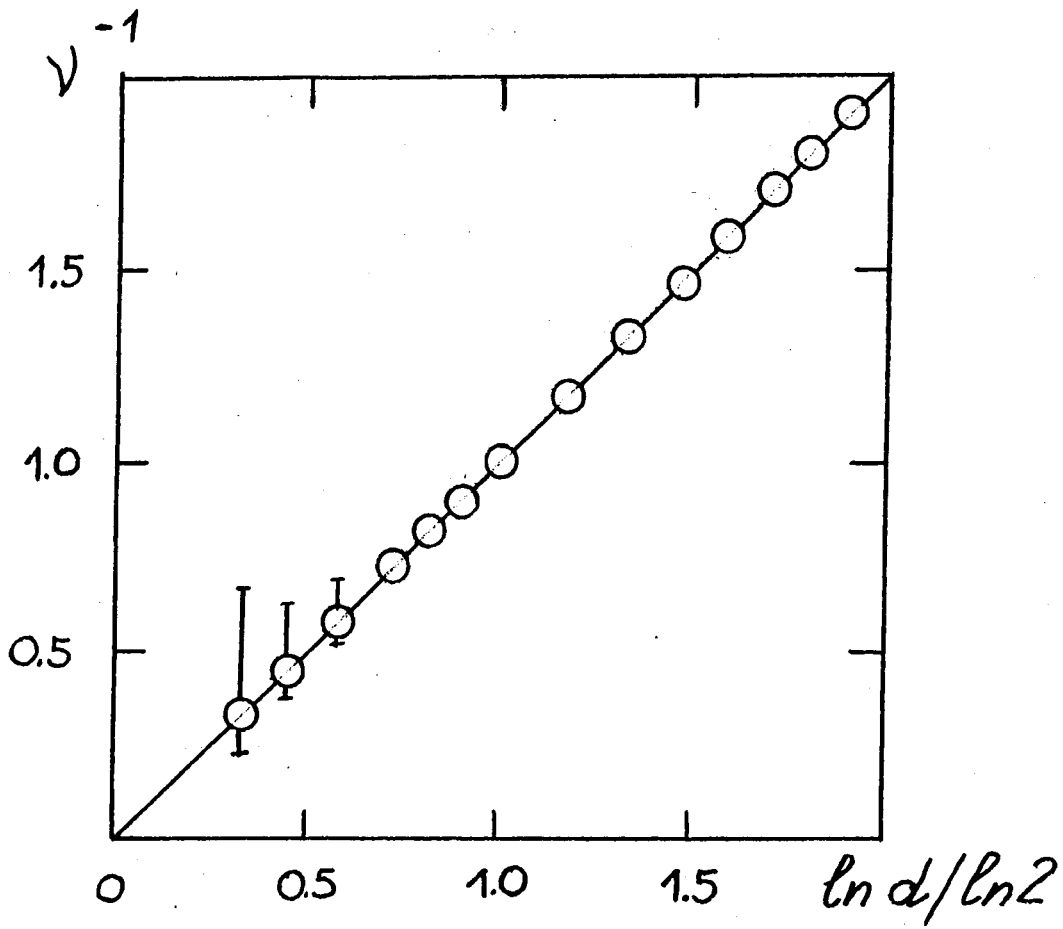

Рис. 1

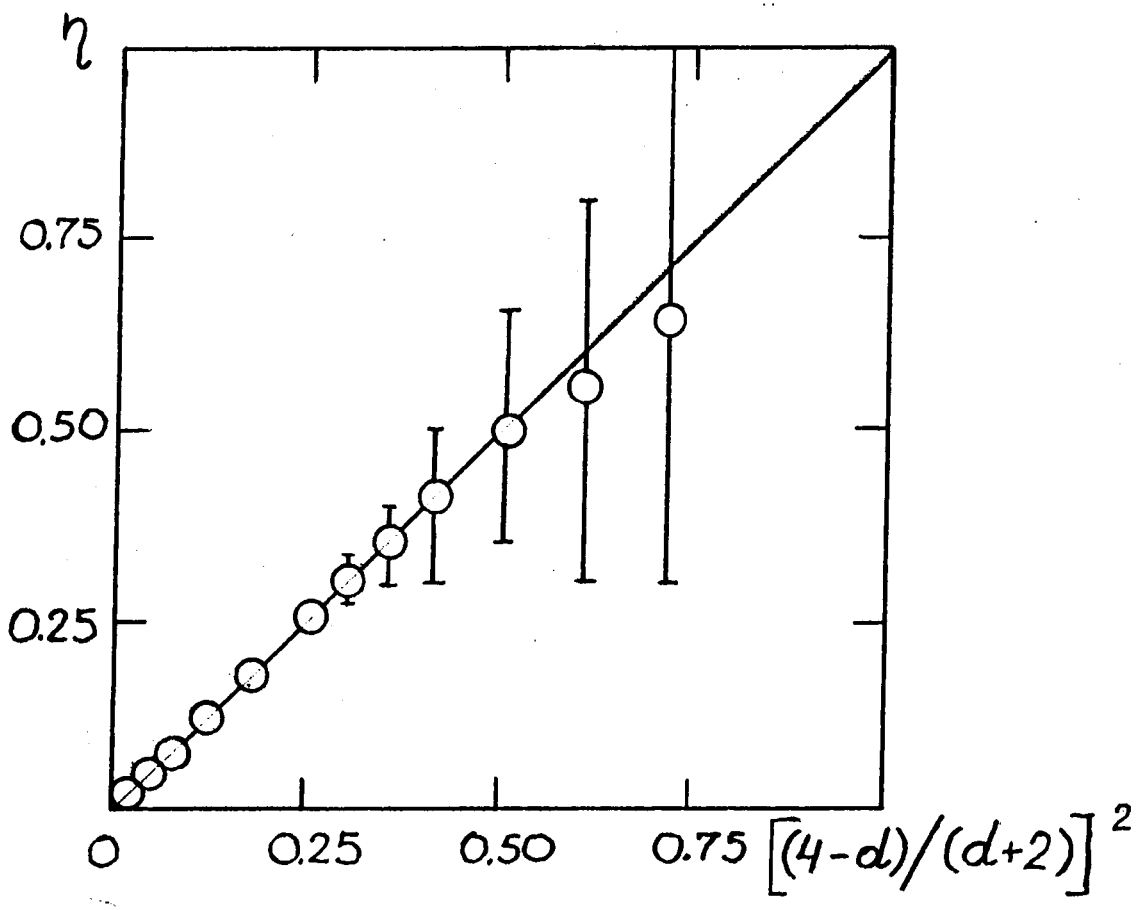

Рис. 2.

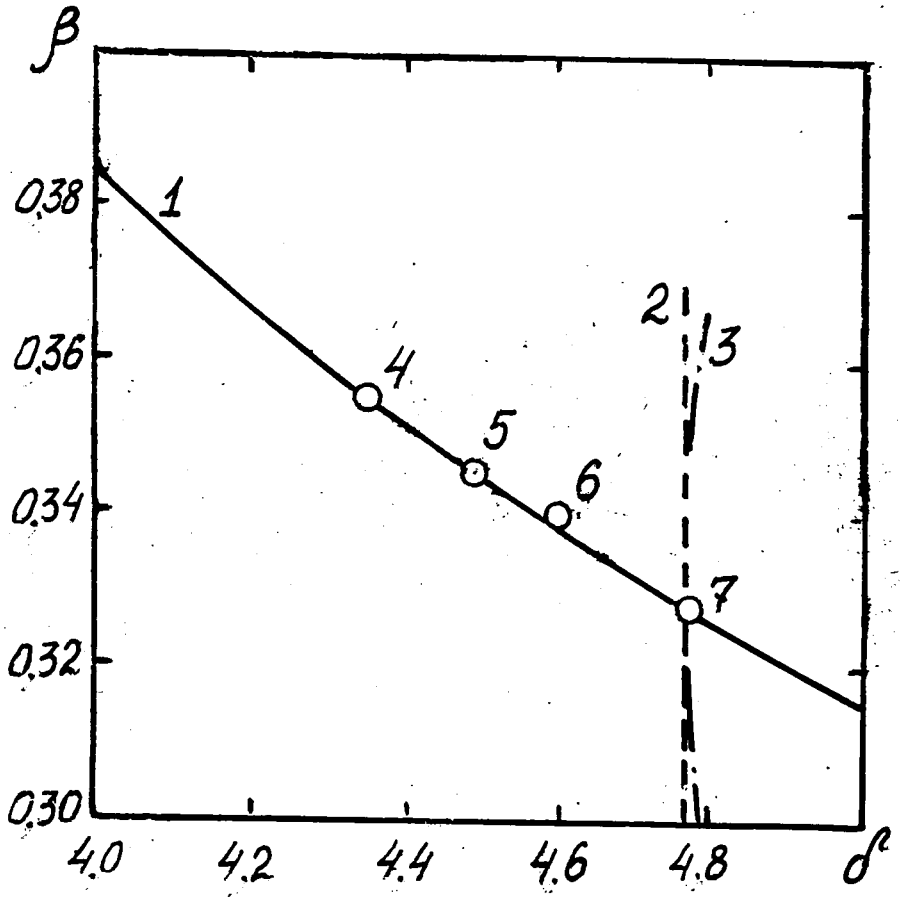

Рис. 3.

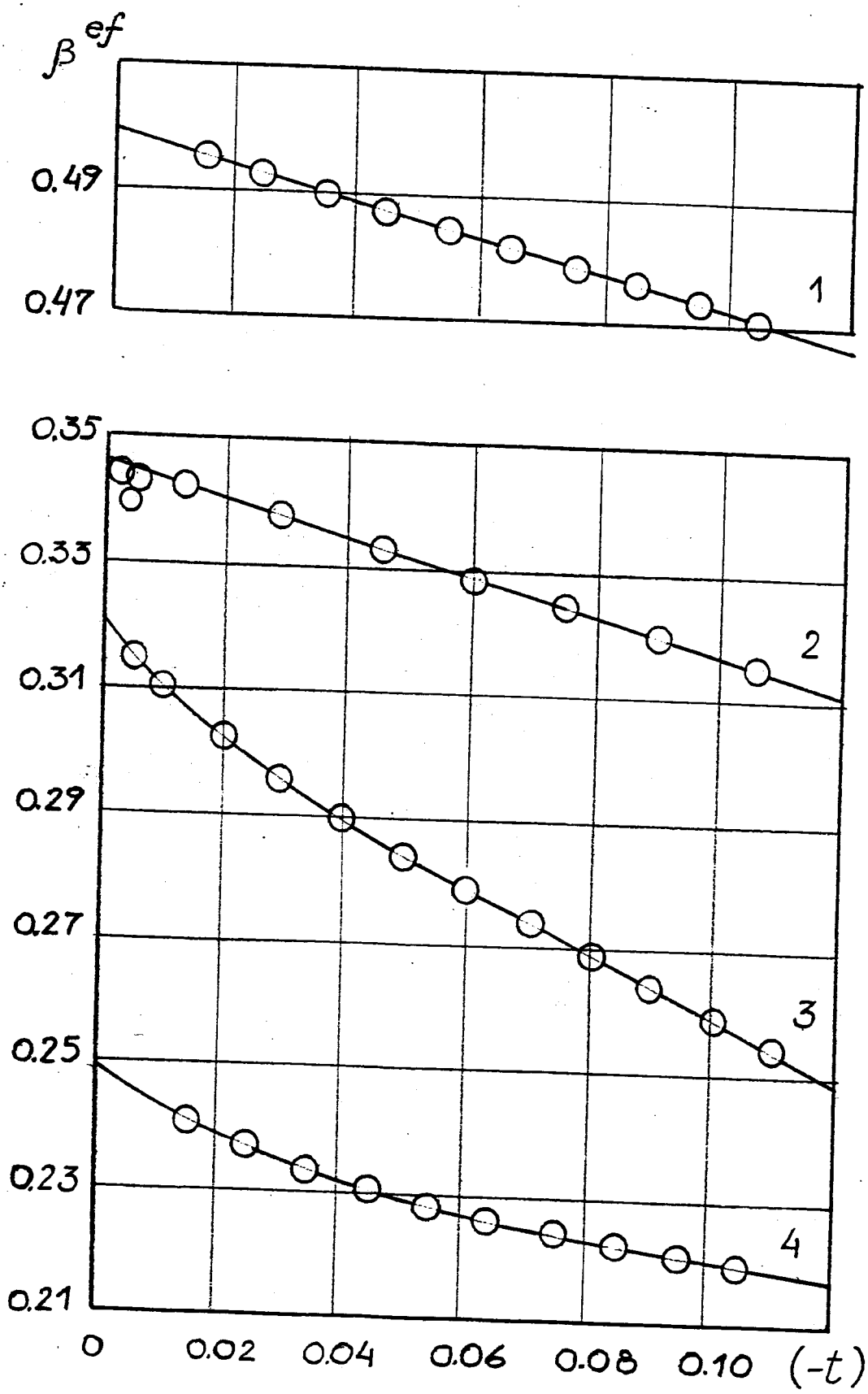
β^{ef}
0.49
0.47
1
0.35
0.33
0.31
0.29
0.27
0.25
0.23
0.21
2
3
4
0
0.02
0.04
0.06
0.08
0.10
(-t)

Рис 4

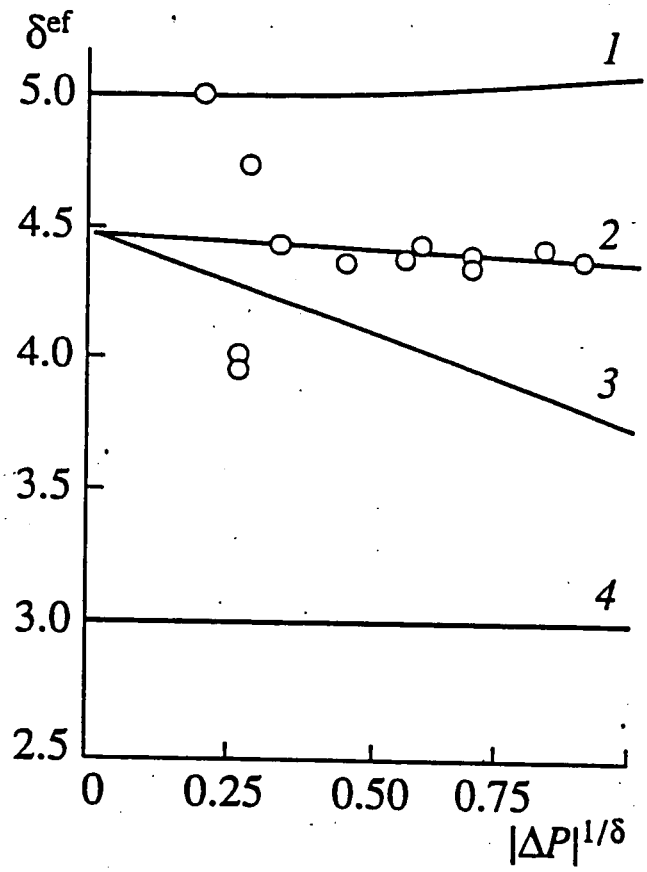

Рис. 5.

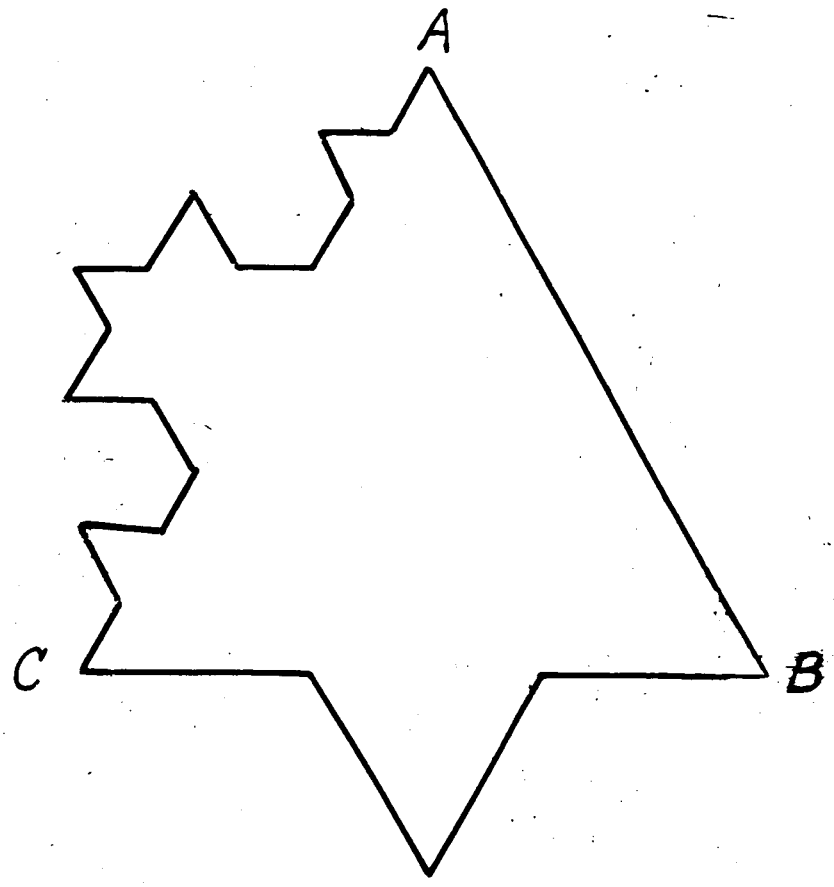

Рис. 6