WSE-142

ON-WAFER NOISE-PARAMETER MEASUREMENTS & UNCERTAINTIES AT NIST

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Mostly done as part of "Kelvin Project," with IBM & RF Micro Devices

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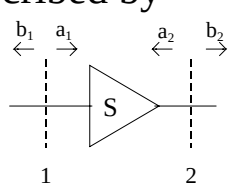
RFMD: T. McKay, G.A. Rezvani, L. Reynolds, J. Tao, M. Forrester

OUTLINE

- Theoretical framework, wave representation of noise matrix
- On-wafer calibration, reference planes, probe corrections
- Measurement method
- Uncertainty analysis
- Results & Checks
- Simulations & possible improvements

FRAMEWORK

- Formalism follows wave representation of noise correlation matrix [1]
- Linear two-port (amp, transistor, attenuator,…) described by



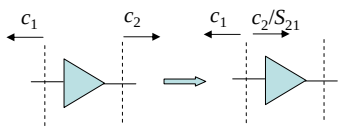
$$\begin{pmatrix} b_1 \\ b_2 \end{pmatrix} = \begin{pmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} + \begin{pmatrix} c_1 \\ c_2 \end{pmatrix}$$

- Intrinsic noise correlation matrix defined by

$$\hat{N}_{ij} = \left\langle c_i c_j^* \right\rangle$$

Normalization: $|c|^2$ = spectral power

- 4 independent parameters: N_{11}, N_{22}, complex N_{12}
- Convenient to define variables referred to input by scaling $c_2 \rightarrow c_2/S_{21}$ [2],



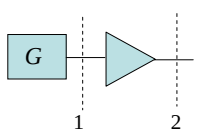
$$k_B X_1 \equiv \left\langle |c_1|^2 \right\rangle = \hat{N}_{11}$$

$$k_B X_2 \equiv \left\langle \left| \frac{c_2}{S_{21}} \right|^2 \right\rangle = \frac{\hat{N}_{22}}{|S_{21}|^2}$$

$$k_B X_{12} \equiv \left\langle c_1 \left(\frac{c_2}{S_{21}} \right)^* \right\rangle = \frac{\hat{N}_{12}}{S_{21}^*}$$

- X's have dimensions of temperature.

- In terms of X's,

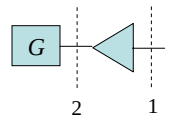


$$T_2 = \frac{|S_{21}|^2}{\left(1-|\Gamma_2|^2\right)} \left\{ \frac{\left(1-|\Gamma_G|^2\right)}{|1-\Gamma_G S_{11}|^2} T_G + \left| \frac{\Gamma_G}{1-\Gamma_G S_{11}} \right|^2 X_1 + X_2 + 2\mathrm{Re}\left[\frac{\Gamma_G X_{12}}{1-\Gamma_G S_{11}} \right] \right\}$$

G_{av}

n.b.: linear fit if just forward configuration.

- Also measure reverse configuration,



$$T_1 = \frac{1}{\left(1-|\Gamma_1|^2\right)}\left\{\frac{\left(1-|\Gamma_G|^2\right)|S_{12}|^2}{|1-\Gamma_G S_{22}|^2}T_G + \left|\frac{S_{12}S_{21}\Gamma_G}{1-\Gamma_G S_{22}}\right|^2 X_2 + X_1 + 2\,\mathrm{Re}\left[\frac{S_{12}S_{21}\Gamma_G X_{12}^*}{1-\Gamma_G S_{22}}\right]\right\}$$

- Can relate X's to IEEE parameters,

$$T_2 = G_{av}\left(T_G + T_e\right) \qquad T_e = T_{\min} + t\frac{|\Gamma_G - \Gamma_{opt}|^2}{\left(1-|\Gamma_G|^2\right)|1+\Gamma_{opt}|^2} \qquad t = \frac{4R_n T_0}{Z_0}$$

X's → IEEE

$$t = X_1 + |1+S_{11}|^2 X_2 - 2\,\mathrm{Re}\left[(1+S_{11})^* X_{12}\right],$$

$$T_{e,\min} = \frac{X_2 - |\Gamma_{opt}|^2\left[X_1 + |S_{11}|^2 X_2 - 2\,\mathrm{Re}\left(S_{11}^* X_{12}\right)\right]}{\left(1+|\Gamma_{opt}|^2\right)},$$

$$\Gamma_{opt} = \frac{\eta}{2}\left(1-\sqrt{1-\frac{4}{|\eta|^2}}\right),$$

$$\eta = \frac{X_2\left(1+|S_{11}|^2\right) + X_1 - 2\,\mathrm{Re}\left(S_{11}^* X_{12}\right)}{\left(X_2 S_{11} - X_{12}\right)}.$$

IEEE → X's

$$X_1 = T_{e,\min}\left(|S_{11}|^2 - 1\right) + \frac{t\left|1-S_{11}\Gamma_{opt}\right|^2}{\left|1+\Gamma_{opt}\right|^2},$$

$$X_2 = T_{e,\min} + \frac{t\left|\Gamma_{opt}\right|^2}{\left|1+\Gamma_{opt}\right|^2},$$

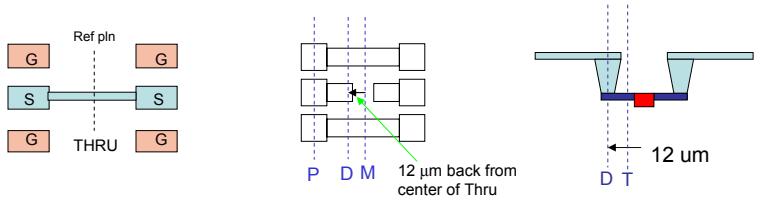
$$X_{12} = S_{11}T_{e,\min} - \frac{t\Gamma_{opt}^*\left(1-S_{11}\Gamma_{opt}\right)}{\left|1+\Gamma_{opt}\right|^2}.$$

Notes: $X_2 = T_{e,0}$

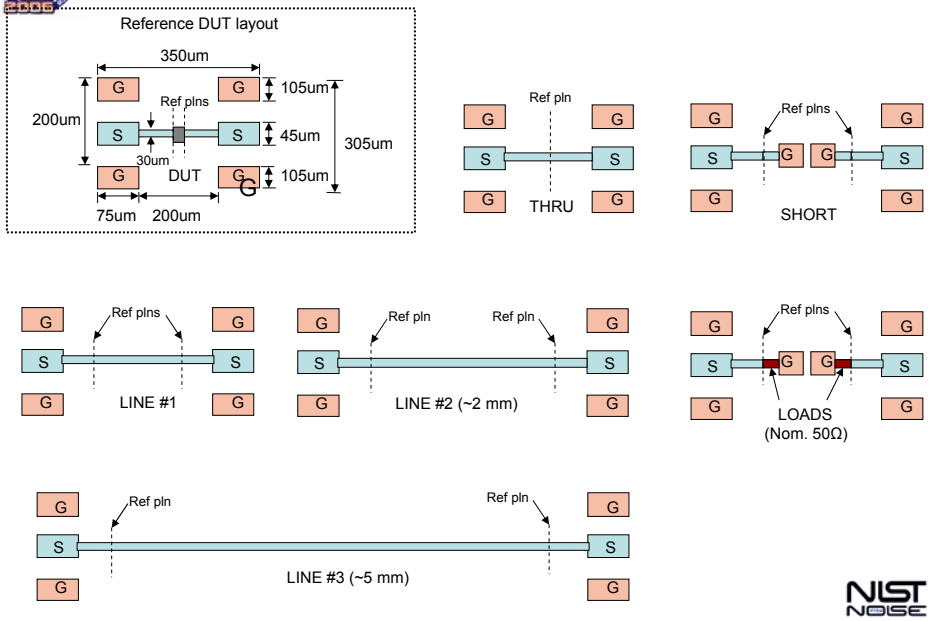
Bound implied by $X_1 \geq 0$

CALIBRATION, REFERENCE PLANES, …

- Use on-wafer multiline TRL calibration [3] with on-wafer standards. (Could use a compact cal set to save real estate.)
- Reference plane defined by center of through (M), can be translated since calibration also characterizes transmission line.



TRL Calibration Set



- Compact cal set (TRL+M) is also possible.
- We do not attempt to deembed down to transistor reference planes (T).
- Use "pseudo-waves," defined with respect to a real reference impedance of 50 Ω. (Allows us to maintain $p_{net} = |a|^2 - |b|^2$, despite lossy lines on wafer).

Noise Measurement

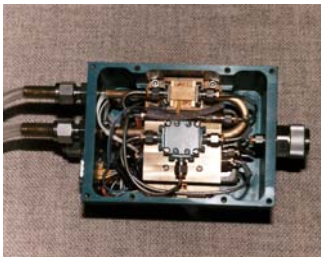
- NIST uses a radiometer-based method similar to its method for packaged amplifiers.
- Ambient & cryogenic (liquid nitrogen) primary standards.

$u_{TCry} \approx 0.65$ K

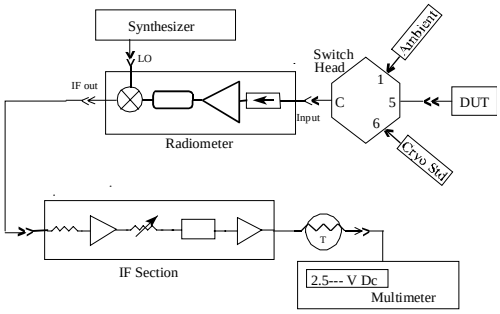
$u_{TAmb} \approx 0.1$ K

Noise Measurement (cont'd)

- Radiometer, switch housing with ambient standard.

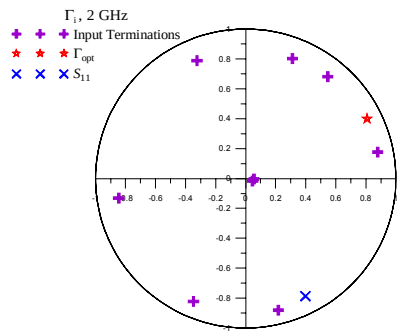


- Total-power radiometer, double sideband, baseband down-conversion, 5 MHz bandwidth (each sideband), isolated, system noise temperature ≈ 450 K at 8 GHz, gain ≈ 100 dB. [4]

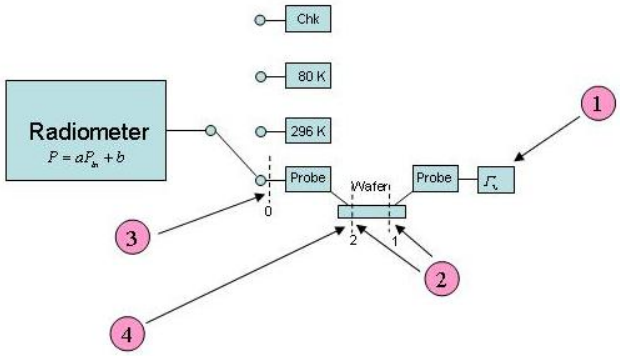


On-Wafer Setup

- S-parameters measured on VNA, noise on radiometer.
- Use discrete terminations on input: slow, painful, but repeatable & flexible. (Choice of input states still under study.)

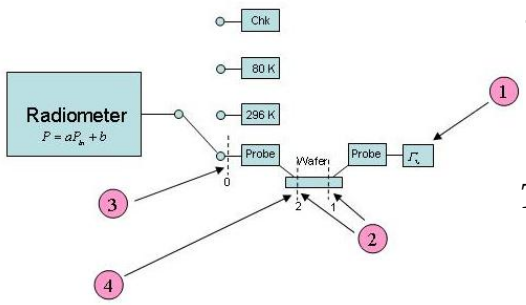


Measurement Procedure



1. Series of different terminations *i*.
2. (a) $\Gamma_{1,i}$, $\Gamma_{2,i}$, and S_{ij} measured with on-wafer cal
   (b) $T_{1,i}$ measured on wafer
3. Measure $T_{0,i}$

Measurement Procedure (cont'd)



4. Compute $T_{2,i}$:

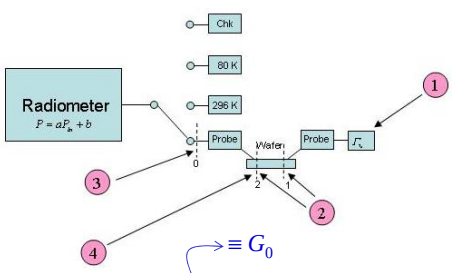
$$T_0 = \alpha_{02} T_2 + (1 - \alpha_{02}) T_{amb}$$

$$T_{2,i} = \frac{T_{0,i} - (1 - \alpha_{02,i}) T_{amb}}{\alpha_{02,i}}$$

where $\alpha_{02,i} \equiv \frac{P_0^{avail}}{P_2^{avail}} = \frac{|S_{21}(02)|^2 \left(1 - |\Gamma_{2,i}|^2\right)}{|1 - \Gamma_{2,i} S_{11}(02)|^2 \left(1 - |\Gamma_{0,i}|^2\right)}$

5. Measure $T_{rev} = T_1$ in reverse configuration,

Measurement Procedure (cont'd)



6. Weighted fit to eqns. for T_2 and T_1.

$$T_{2,i} = \frac{\overbrace{|S_{21}|^2}^{\equiv G_0}}{\left(1 - |\Gamma_{2,i}|^2\right)} \left\{ \frac{\left(1 - |\Gamma_{1,i}|^2\right)}{|1 - \Gamma_{1,i} S_{11}|^2} T_{1,i} + \left| \frac{\Gamma_{1,i}}{1 - \Gamma_{1,i} S_{11}} \right|^2 X_1 + X_2 + 2\,\mathrm{Re}\left[\frac{\Gamma_{1,i} X_{12}}{1 - \Gamma_{1,i} S_{11}} \right] \right\}$$

$$T_1(rev.config) = \frac{1}{\left(1 - |\Gamma_1|^2\right)} \left\{ \frac{|S_{12}|^2 \left(1 - |\Gamma_{2,L}|^2\right)}{|1 - \Gamma_{2,L} S_{22}|^2} T_{2,L} + \left| \frac{S_{12} S_{21} \Gamma_{2,L}}{1 - \Gamma_{2,L} S_{22}} \right|^2 X_2 + X_1 + 2\,\mathrm{Re}\left[\frac{S_{12} S_{21} \Gamma_{2,L} X_{12}^*}{1 - \Gamma_{2,L} S_{22}} \right] \right\}$$

UNCERTAINTIES

- Follow ISO Guide to Uncertainty in Measurement (GUM) [5]
- Type A (statistical): obtained in the fitting process, from covariance matrix V_{ij}

$$u_A(i) = \sqrt{V_{ii}}$$

- Fit is done for X's, so the type-A uncertainties are for the X's. To get type-A uncertainties for the IEEE parameters,

$$u_i(IEEE) = \sqrt{V_{ii}(IEEE)}$$

$$V_{ij}(IEEE) = \sum_{i',j'=1}^{5} D_{ii'} D_{jj'} V_{i'j'}(X's)$$

$$D = \begin{pmatrix} \frac{\partial T_{\min}}{\partial X_1} & \frac{\partial T_{\min}}{\partial X_2} & \frac{\partial T_{\min}}{\partial \operatorname{Re} X_{12}} & \frac{\partial T_{\min}}{\partial \operatorname{Im} X_{12}} & 0 \\ \frac{\partial t}{\partial X_1} & \frac{\partial t}{\partial X_2} & \frac{\partial t}{\partial \operatorname{Re} X_{12}} & \frac{\partial t}{\partial \operatorname{Im} X_{12}} & 0 \\ \frac{\partial \operatorname{Re} \Gamma_{opt}}{\partial X_1} & \dots & \dots & \dots & 0 \\ \frac{\partial \operatorname{Im} \Gamma_{opt}}{\partial X_2} & \dots & \dots & \dots & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

- Type-B uncertainties are all other uncertainties, i.e., not evaluated by statistical means.
- We "know" uncertainties in underlying quantities ($T_{G,i}$, $\Gamma_{G,i}$, $T_{out,i}$, S, T_{amb} , …); want the resulting uncertainties in noise parameters.
- Estimate them with a Monte Carlo program [2,6]
  - use measured values as hypothetical "true" values
  - input uncertainties (& distributions) in reflection coefficients, noise temperature of non-ambient source, ambient temperature, measurement of output noise temperature (or power), correlations, ...

- MC Program (cont'd)
  - generate set of simulated measurement data for $T_{G,i}$, $\Gamma_{G,i}$, $T_{out,i}$, S, and T_{amb} , *e.g.*, $T_{meas} = T_{true} + \varepsilon_T$ where $< \varepsilon_T > = 0$, $< \varepsilon_T^2 > = u_T^2$
  - analyze simulated data as if it were real data, compute the "measured" noise parameters & G
  - discard "bad" data sets
  - repeat (simulate, analyze, repeat)
  - compute type-B uncertainties,

$$u(y) \approx RMSE(y) = \sqrt{Var(y) + (\bar{y} - y_{true})^2} \ .$$

- Standard (combined) uncertainty: $u_c = \sqrt{u_A^2 + u_B^2}$

- Values used for underlying uncertainties:

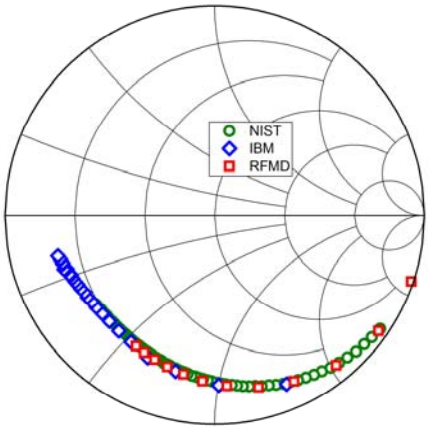
	σ_{cor}	σ_{uncor}
$\Gamma_{G,i} \leq 0.005$:	0.003	0.004
$\Gamma_{G,i} > 0.005$:	0.003	0.004
S_{21}:	0.003	0.004
T_{amb} :	0.0	0.5 K (rect. distr.)
$T_{in,hot}$:		1 % [7]
$T_{out,meas}$:	0.8 %	0.6 %

- Will see resulting uncertainties in noise parameters below.

SOME RESULTS

- Measurements & comparisons done as part of "Kelvin Project," with IBM & RF Micro Devices (RFMD) [8,9]
- 128×3×0.12 NMOS device
  - 128 fingers of polysilicon over
  - 3 μm wide active channel
  - 0.12 μm gate length
  - fabricated in 0.13 μm technology (by IBM)
- Bias:
  - drain voltage V_{ds} = 1.2 V
  - J = 25 μA/μm

S Parameters

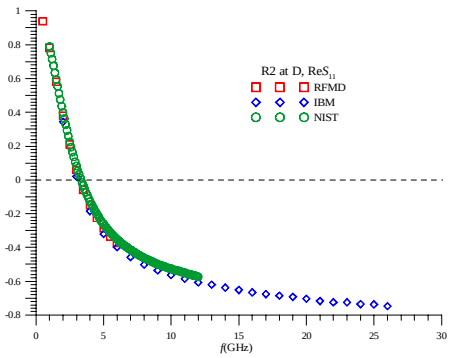


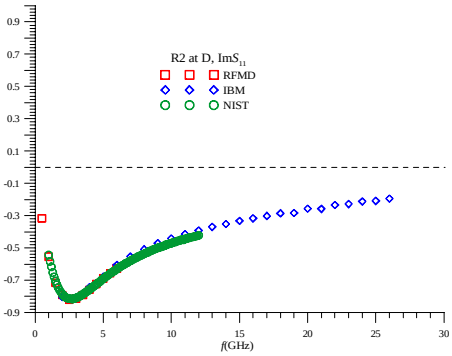
S_{11} at plane D

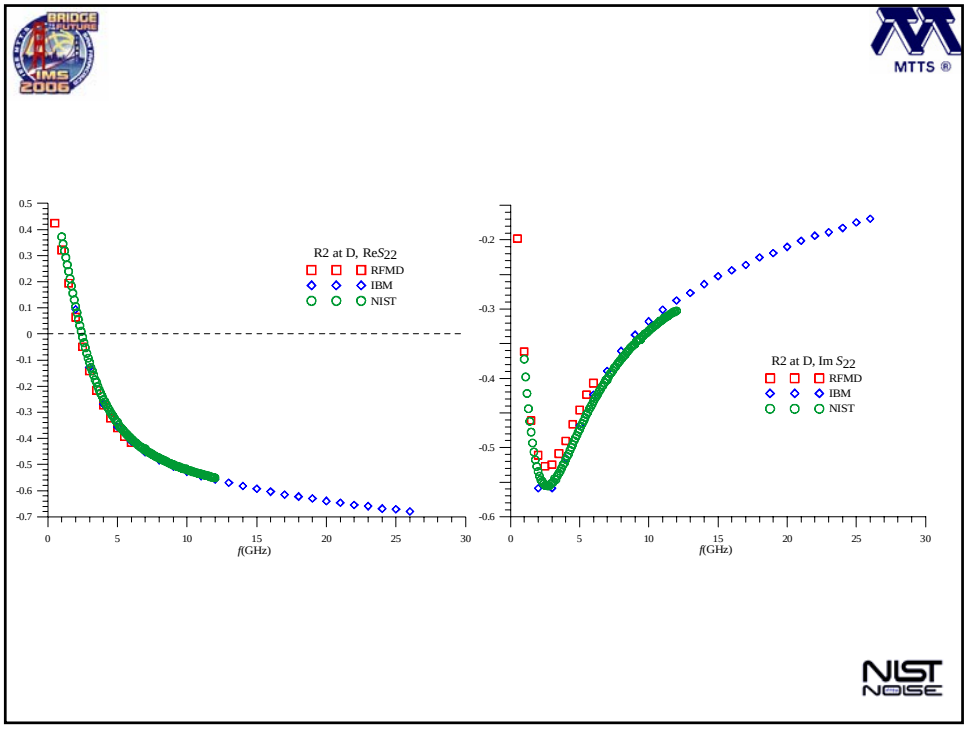
Generally agree well,
but some small differences.

Frequency ranges

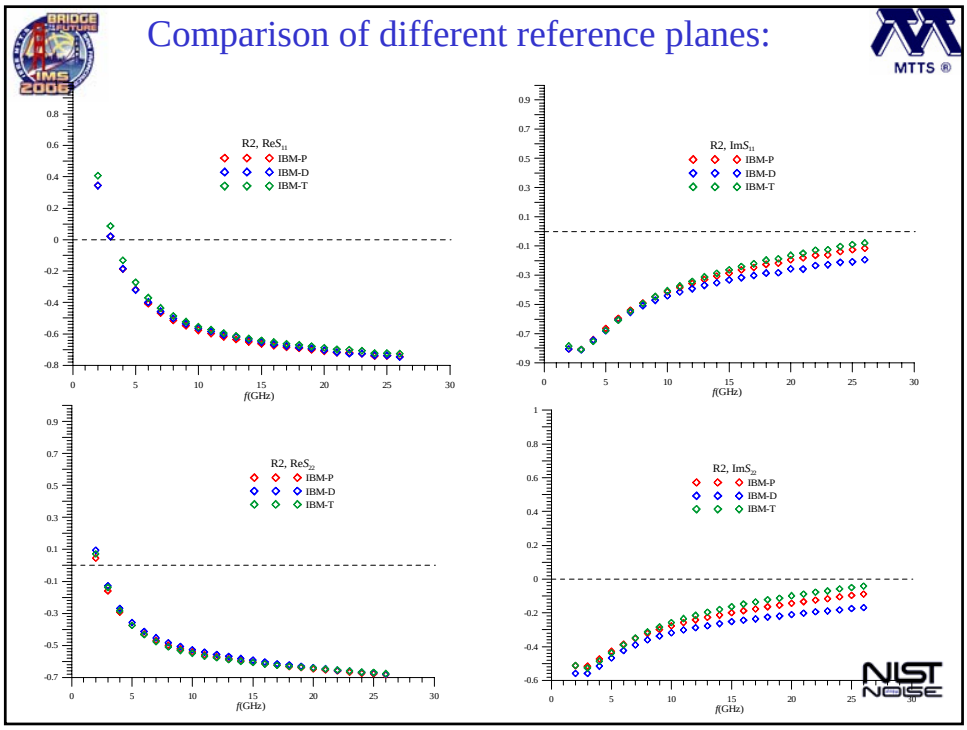
RFMD 0.5 – ~~6~~ 26 GHz (to cover cellular bands)
IBM: 2 – 26 GHz
NIST: 1 – 12 GHz



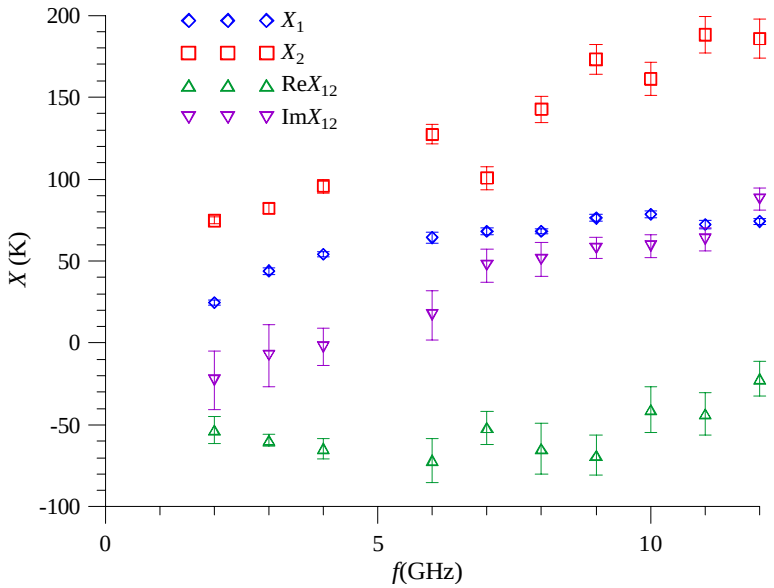




Comparison of different reference planes:

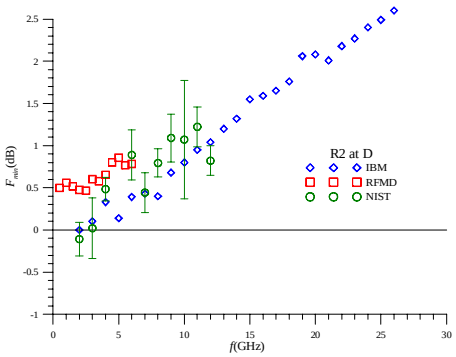


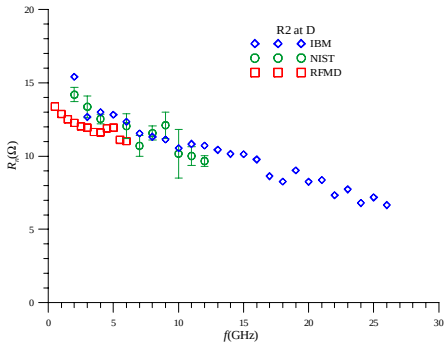
Noise Parameters (at D): *X*'s



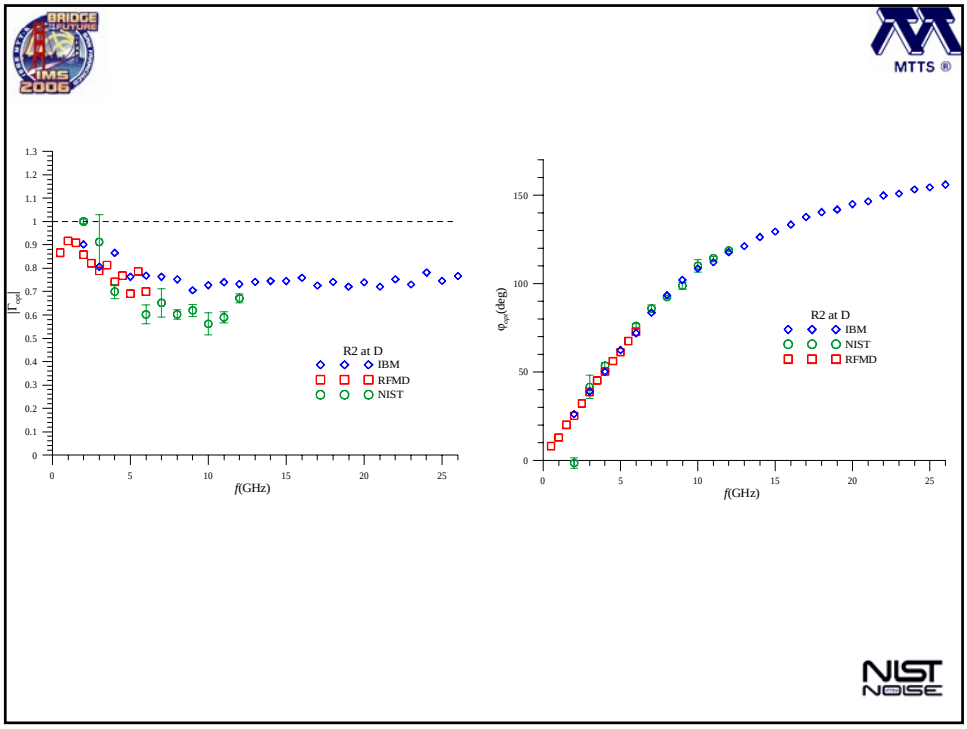
Noise Parameters (at D):

All labs used TRL cal at D.

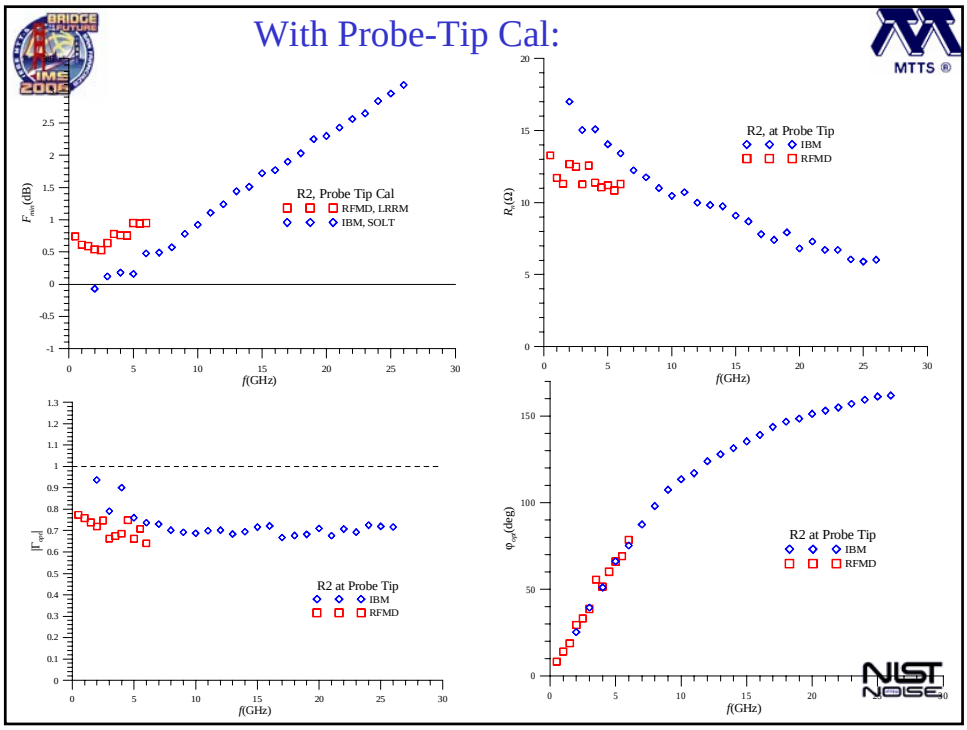




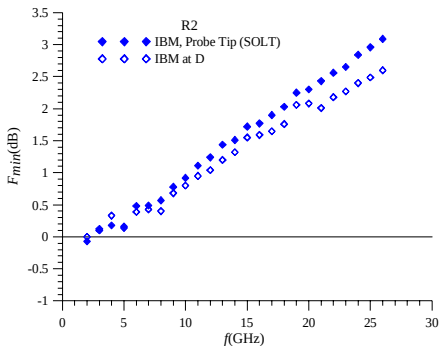
Results are probably consistent within expected uncertainties, but it is clear that the device performance is better than our ability to measure it.

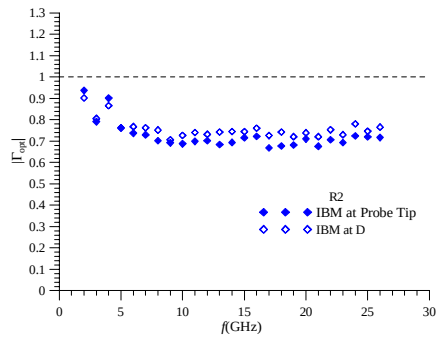


With Probe-Tip Cal:



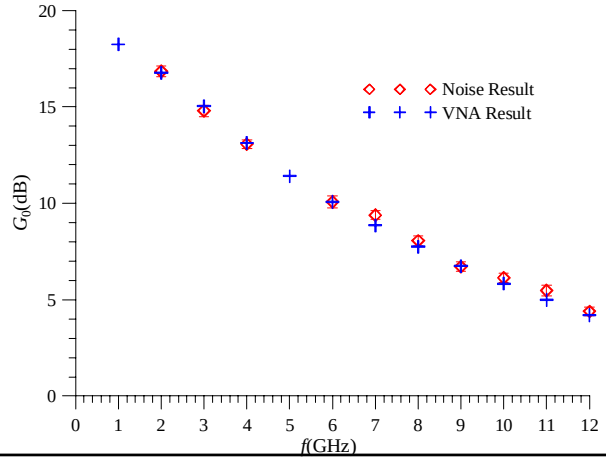
Different Reference Planes:





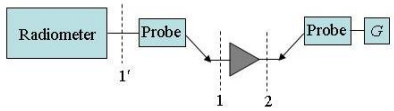
CHECKS

- Get $G_0 \equiv |S_{21}|^2$ from the fit; also measure it (independently) with VNA. So compare the two sets of results.



T_1 Check

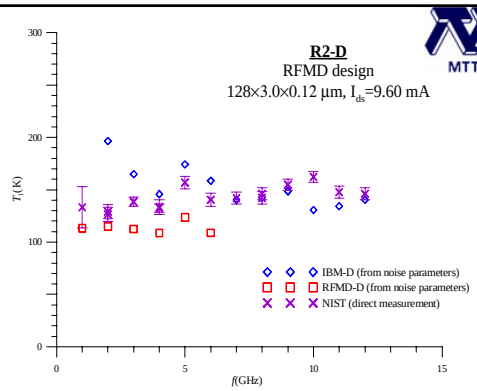
- Can directly measure reverse noise T_1

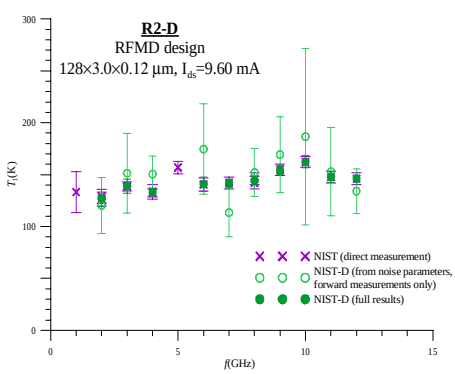


- And can compute T_1 from noise parameters

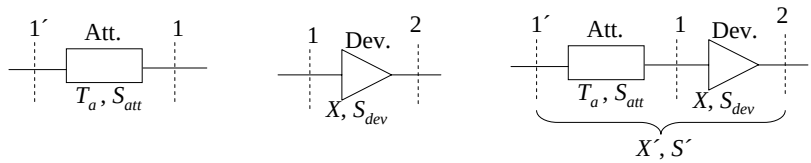
$$T_1 = \frac{1}{\left(1-|\Gamma_1|^2\right)}\left\{\frac{\left(1-|\Gamma_G|^2\right)|S_{12}|^2}{|1-\Gamma_G S_{22}|^2}T_G + \left|\frac{S_{12}S_{21}\Gamma_G}{1-\Gamma_G S_{22}}\right|^2 X_2 + X_1 + 2\,\mathrm{Re}\left[\frac{S_{12}S_{21}\Gamma_G X_{12}^*}{1-\Gamma_G S_{22}}\right]\right\}$$

- So do both & compare [9]





Cascade Test



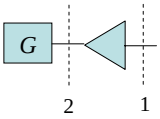
- Measure S_{att}, X, S_{dev} ; predict $X´$& compare.
- Have used this test with an isolator for connectorized amplifiers; very successful. [10]
- About to use it on-wafer with an attenuator & transistor.

SIMULATIONS & POSSIBLE IMPROVEMENTS

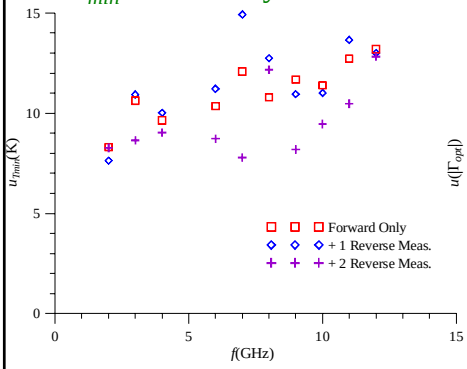
- Obvious possibility is cal of hot noise source.
- Can also use the Monte Carlo uncertainty program to test possible improvements.
- Caution: results are for NIST methods & system. Expect similar results for other systems, but …
- "Plan" to extend program to more common or more general systems & methods.
- Consider two possible improvements here:
  - Inclusion of one or more reverse noise measurements.[9]
  - Use of a cold (i.e., significantly below ambient) input noise source.

Reverse Noise Measurement(s)

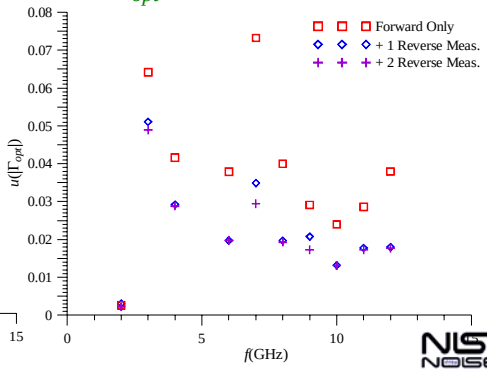
- Inclusion of one or more reverse noise measurements improves uncertainties, especially for $|\Gamma_{opt}|$



T_{min} Uncertainty

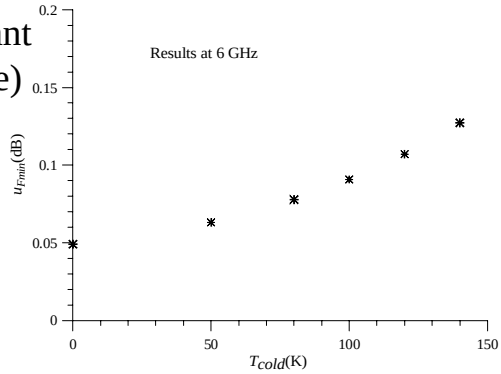


$|\Gamma_{opt}|$ Uncertainty

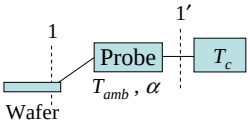


Cold Input Noise Source

- For maximum effect, want T_{cold} as low (and accurate) as possible.



- Probe causes problems:
  $T_1 = \alpha T_{1'} + (1 - \alpha)T_{amb}$

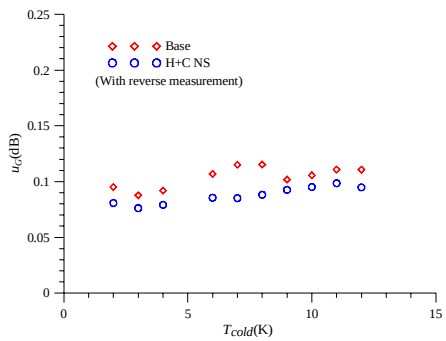


- If probe has 1 dB loss, $(1 - \alpha)T_{amb} \approx 62$ K at $T_{amb} = 296$ K.

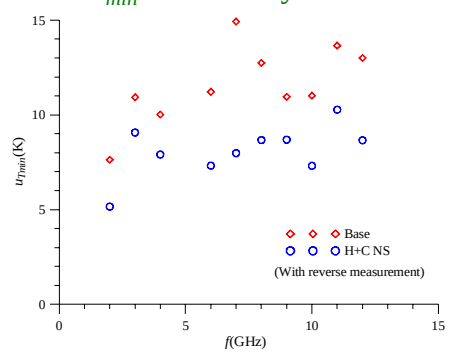
Cold Input Noise Source (cont'd)

- With reasonable, "good" values for α, T_{cold}, u_{Tcold}, etc., significant improvement if use cold source *in addition to* (not instead of) hot source.

G Uncertainty



T_{min} Uncertainty



($u_{Fmin} \approx 0.15$ dB $\rightarrow$ 0.10 dB)

SUMMARY

- We measure the noise parameters at an on-wafer reference plane, but do not deembed to the transistor.
- Noise performance of the devices we measure (0.12 µm gate length NMOS) is better than our (and probably your) ability to measure it.
- We believe we have ways to improve the measurement techniques.

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NIST Noise publications & presentation slides available at

http://boulder.nist.gov/div818/81801/Noise/index.html