NOISE-PARAMETER MEASUREMENTS AT NIST

J. Randa
D.K. Walker & R.L. Billinger
J. Bridges & G. Free

Boulder, 10/28/04

Noise Project Areas

- Noise temperature of (1-port) noise sources
- Noise parameters of 2-ports (amplifiers & transistors)
- Remote-sensing radiometer calibration

Why Noise Parameters?

- Signal detection, S/N
- Bandwidth
- Power requirements
- …….

Outline

I. Background
- noise temperature
- noise figure
- noise parameters, IEEE formulation
- measurement

II. Wave Representation & Noise Matrix
- noise matrix
- noise parameters
- measurement method

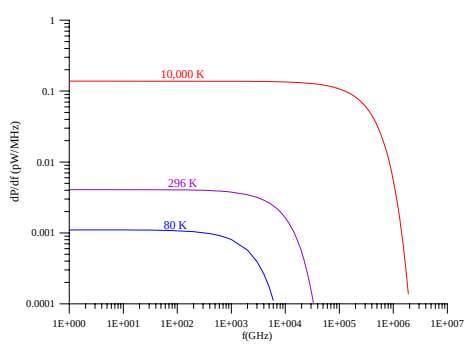
III. Measurements
- terminations & switching
- checks
- measurement results
- uncertainties

IV. On-Wafer Noise Parameters

I. Background

Noise Temperature

- For passive device, at physical temperature T, with small Δf,

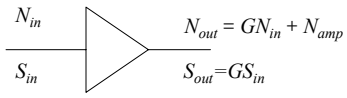


$$\langle P_{avail}(f)\rangle = \frac{hf}{e^{hf/(k_B T)}-1}\Delta f$$

Note: *very* small powers

- small f: $\langle P_{avail} \rangle \approx k_B T \Delta f [1 - hf/(2k_B T)]$
  $\approx k_B T \Delta f$
- What about active devices? Can we define a noise temperature?
- Several different definitions in use.
- Most sensible (*i.e.*, our) choice is
  noise temp ≡ available spectral noise-power density divided by Boltzmann's constant.
- It is the common choice in international comparisons and elsewhere.
- It is much more convenient for amplifier noise considerations (at least for careful ones)

Noise Figure



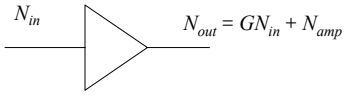
N and S are spectral power densities, $N = k_B T$

- $$\frac{(S/N)_{in}}{(S/N)_{out}} = \frac{S_{in}}{GS_{in}} \frac{(GN_{in} + N_{amp})}{N_{in}} = \frac{OutputNoise}{G \times InputNoise}$$

- Noise Figure ≡ (Noise out)/(Noise out due to Noise in), evaluated for $T_{in} = T_0 \equiv 290$ K. ($N_{in} = k_B T_0$)
- In terms of noise temperatures, let $N_{amp} = Gk_B T_e$. Then

$$NF = \frac{Gk_B T_0 + Gk_B T_e}{Gk_B T_0} = \frac{(T_0 + T_e)}{T_0}$$

Note regarding noise-temperature definition:



If $N = kT$, then $N_{in} = kT_{in}$, etc., and $T_{out} = GT_{in} + GT_e$

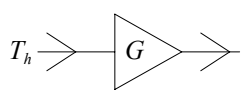
But if we use the "equivalent physical temperature" definition, then $N_{in} = \dfrac{hf}{e^{hf/kT_{in}} - 1}$

and similarly for the others, and so $\dfrac{hf}{e^{hf/kT_{out}} - 1} = G\left(\dfrac{hf}{e^{hf/kT_{in}} - 1} + \dfrac{hf}{e^{hf/kT_e} - 1}\right)$.

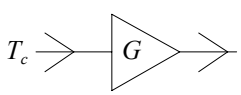
Solving for T_{out} , we would get

$$T_{out} = \frac{hf}{k}\left\{\ln\left[1 + \frac{1}{G}\left(\frac{1}{\left(e^{hf/kT_{in}} - 1\right)} + \frac{1}{\left(e^{hf/kT_e} - 1\right)}\right)^{-1}\right]\right\}^{-1}$$

Simple-Case Measurement, all Γ's equal



$N_{out,h} = Gk_B(T_h + T_e)$



$N_{out,c} = Gk_B(T_c + T_e)$

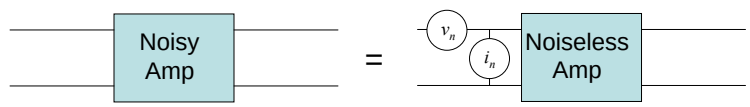
Combine & solve:

$$G = \frac{N_h - N_c}{k_B(T_h - T_c)} \qquad T_e = \frac{N_c T_h - N_h T_c}{N_h - N_c} = \frac{T_h - YT_c}{Y - 1} \qquad \text{where } Y = N_h/N_c$$

$$NF = 1 + \frac{T_e}{T_0} = 1 + \frac{T_h - YT_c}{(Y-1)T_0}$$

Noise Parameters, IEEE Representation

- Equivalent circuit:



- (Noise out)/(Noise in) depends on impedance of input termination, $NF = NF(Z_S)$ or $NF(\Gamma_S)$, & $T_e = T_e(Z_S$ or $\Gamma_S)$,

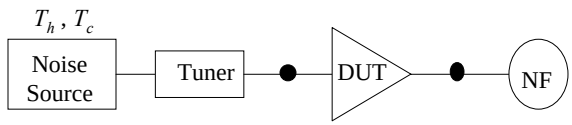
$$NF = NF_{\min} + \frac{4R_n}{Z_0}\frac{\left|\Gamma_{opt} - \Gamma_S\right|^2}{\left|1+\Gamma_{opt}\right|^2\left(1-\left|\Gamma_S\right|^2\right)} \qquad T_e = T_{e,\min} + t\frac{\left|\Gamma_{opt} - \Gamma_S\right|^2}{\left|1+\Gamma_{opt}\right|^2\left(1-\left|\Gamma_S\right|^2\right)}$$

4 parameters: $T_{e,min}$, $t = 4R_nT_0/Z_0$, and complex Γ_{opt} .

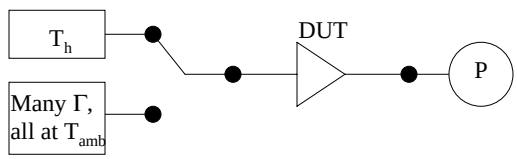
Measuring Noise Parameters

- Many different methods, most based on IEEE parameterization.
- Basic idea of (almost) all methods is to
  - present amplifier (or device) with a variety of different input terminations (Γ & T),
  - have an equation for the "output" in terms of the noise parameters and known quantities (Γ's, T's, S-parameters),
  - determine noise parameters by a fit to the measured output.
  - Need good distrib. of Γ's in complex plane.

- "Output" can be
  - Noise figure



  - Power

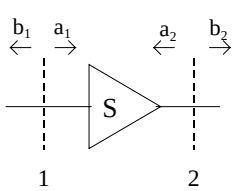


  - Note: output Γ, matching, available power, etc.

II. Wave Representation & Noise Matrix

Noise Matrix

- Linear 2-port:



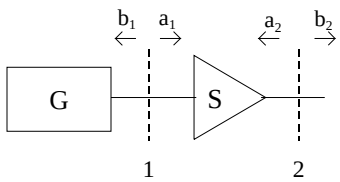
$$\begin{pmatrix} b_1 \\ b_2 \end{pmatrix} = \begin{pmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} + \begin{pmatrix} \hat{b}_1 \\ \hat{b}_2 \end{pmatrix}$$

- Noise matrix: $N_{ij} = \left\langle b_i b_j^* \right\rangle$

  or intrinsic noise matrix: $\hat{N}_{ij} = \left\langle \hat{b}_i \hat{b}_j^* \right\rangle$ 2×2 hermitian matrix, 4 parameters

Noise Parameters

- Output noise temperature T_2



$$k_B T_2 = \frac{|S_{21}|^2}{\left(1-|\Gamma_{GS}|^2\right)}\left[N_G + N_1 + N_2 + N_{12}\right]$$

$$N_G = \frac{\left(1-|\Gamma_G|^2\right)}{|1-\Gamma_G S_{11}|^2} k_B T_G$$

$$N_1 = \left|\frac{\Gamma_G}{1-\Gamma_G S_{11}}\right|^2 \left\langle \left|\hat{b}_1\right|^2 \right\rangle$$

$$N_2 = \left\langle \left|\hat{b}_2 / S_{21}\right|^2 \right\rangle$$

$$N_{12} = 2\,\mathrm{Re}\left[\frac{\Gamma_G}{(1-\Gamma_G S_{11})}\left\langle \hat{b}_1 (\hat{b}_2 / S_{21})^* \right\rangle\right]$$

- So for T_e we have

$$T_e = \frac{|\Gamma_G|^2}{(1-|\Gamma_G|^2)} X_1 + \frac{|1-\Gamma_G S_{11}|^2}{(1-|\Gamma_G|^2)} X_2 + \frac{2}{(1-|\Gamma_G|^2)}\mathrm{Re}\left[(1-\Gamma_G S_{11})^* \Gamma_G X_{12}\right]$$

where $k_B X_1 \equiv \left\langle \left|\hat{b}_1\right|^2 \right\rangle, \quad k_B X_2 \equiv \left\langle \left|\hat{b}_2 / S_{21}\right|^2 \right\rangle, \quad k_B X_{12} \equiv \left\langle \hat{b}_1 \left(\hat{b}_2 / S_{21}\right)^* \right\rangle$

Noise Parameters

- Compare to IEEE parameterization

$$T_e = T_{e,\min} + t\,\frac{\left|\Gamma_G - \Gamma_{opt}\right|^2}{\left(1-|\Gamma_G|^2\right)\left|1+\Gamma_{opt}\right|^2}$$

- General relationships:

X's → IEEE

$$t = X_1 + |1+S_{11}|^2 X_2 - 2\,\mathrm{Re}\left[(1+S_{11})^* X_{12}\right],$$

$$T_{e,\min} = \frac{X_2 - |\Gamma_{opt}|^2\left[X_1 + |S_{11}|^2 X_2 - 2\,\mathrm{Re}\left(S_{11}^* X_{12}\right)\right]}{\left(1+|\Gamma_{opt}|^2\right)},$$

$$\Gamma_{opt} = \frac{\eta}{2}\left(1 - \sqrt{1 - \frac{4}{|\eta|^2}}\right),$$

$$\eta = \frac{X_2\left(1+|S_{11}|^2\right) + X_1 - 2\,\mathrm{Re}\left(S_{11}^* X_{12}\right)}{\left(X_2 S_{11} - X_{12}\right)}.$$

IEEE → X's

$$X_1 = T_{e,\min}\left(|S_{11}|^2 - 1\right) + \frac{t\left|1 - S_{11}\Gamma_{opt}\right|^2}{\left|1+\Gamma_{opt}\right|^2},$$

$$X_2 = T_{e,\min} + \frac{t\left|\Gamma_{opt}\right|^2}{\left|1+\Gamma_{opt}\right|^2},$$

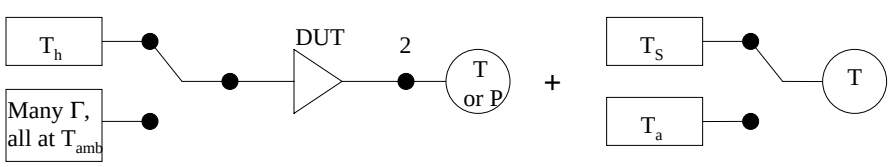
$$X_{12} = S_{11}T_{e,\min} - \frac{t\Gamma_{opt}^*\left(1 - S_{11}\Gamma_{opt}\right)}{\left|1+\Gamma_{opt}\right|^2}.$$

n.b. $X_2 = T_{e,0}$

$X_1 \geq 0$

Measurement

- Noise-matrix approach to measuring noise parameters:



$$k_B T_2 = \frac{|S_{21}|^2}{\left(1 - |\Gamma_{GS}|^2\right)}\left[N_G + N_1 + N_2 + N_{12}\right]$$

$$\Gamma_{GS} = S_{22} + \frac{\Gamma_G S_{12} S_{21}}{(1 - \Gamma_G S_{11})}$$

$$N_G = \frac{\left(1 - |\Gamma_G|^2\right)}{|1 - \Gamma_G S_{11}|^2} k_B T_G$$

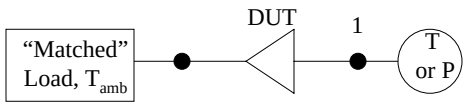
$$N_1 = \left|\frac{\Gamma_G}{1 - \Gamma_G S_{11}}\right|^2 k_B X_1$$

$$N_2 = k_B X_2$$

$$N_{12} = 2\,\mathrm{Re}\left[\frac{\Gamma_G}{(1 - \Gamma_G S_{11})} k_B X_{12}\right]$$

Note: easy to linearize for fit.

- Supplemental measurement, T_{rev}



$$k_B T_1 = \frac{1}{\left(1 - \left|\Gamma_{GS}'\right|\right)}\left[N_G + N_1 + N_2 + N_{12}\right]$$

$$\Gamma_{GS}' = S_{11} + \frac{\Gamma_G S_{12} S_{21}}{(1 - \Gamma_G S_{22})}$$

$$N_G = \frac{|S_{12}|^2\left(1 - |\Gamma_G|^2\right)}{|1 - \Gamma_G S_{22}|^2} k_B T_{amb}$$

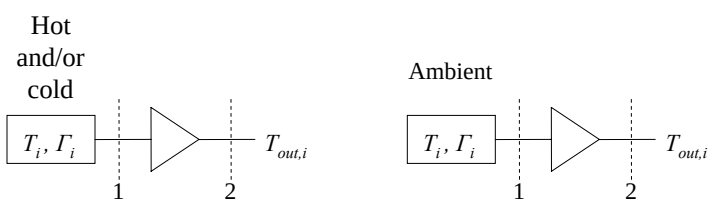
$$N_1 = k_B X_1$$

$$N_2 = \left|\frac{S_{12} S_{21} \Gamma_G}{1 - \Gamma_G S_{22}}\right|^2 k_B X_2$$

$$N_{12} = 2\,\mathrm{Re}\left[\frac{S_{12} S_{21} \Gamma_G}{(1 - \Gamma_G S_{22})} k_B X_{12}^*\right]$$

III. Measurements

Method



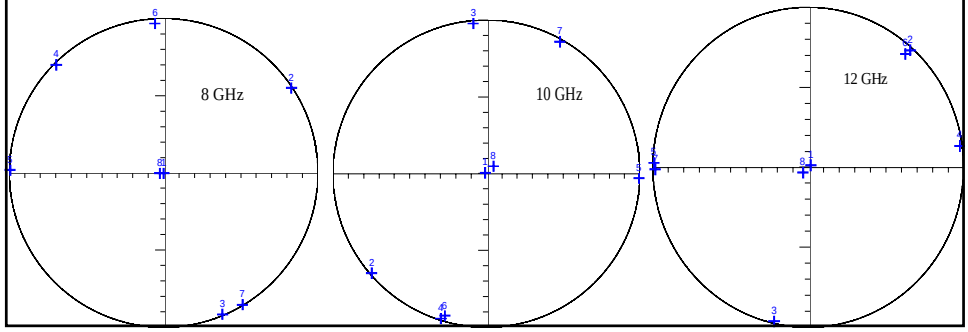
$T_{out} = T_{out}(\Gamma_i, T_i, S_{ij}, X_1, X_2, X_{12})$

Measure for several (1 hot + 7 ambient) terminations and fit for X_1, X_2, X_{12} , and G_0.

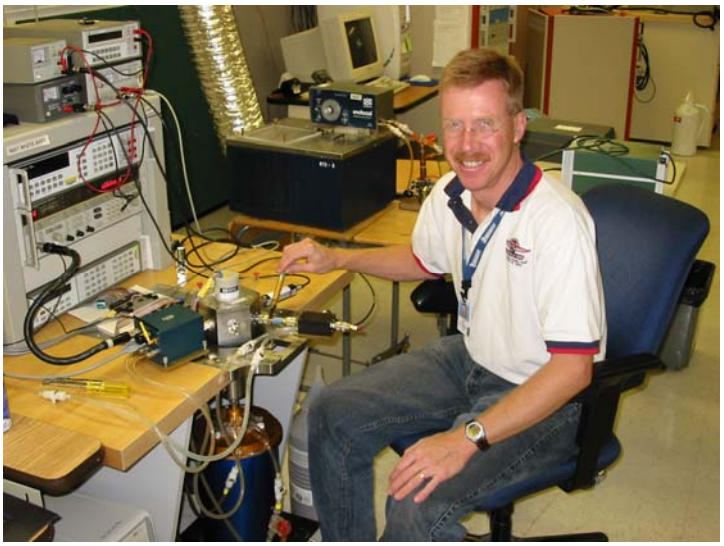
$$G_0 \equiv \frac{|S_{21}|^2}{\left(1 - |S_{11}|^2\right)}$$

Terminations & Switching

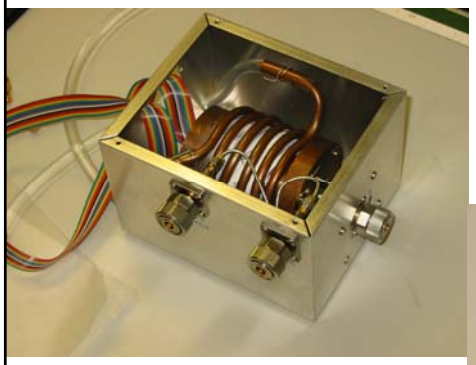
- Tuner possible, but …
- Basic set of terminations:
  1 hot source (~1200 K), matched
  7 ambient-temperature terminations,
  1 matched, others reflective.

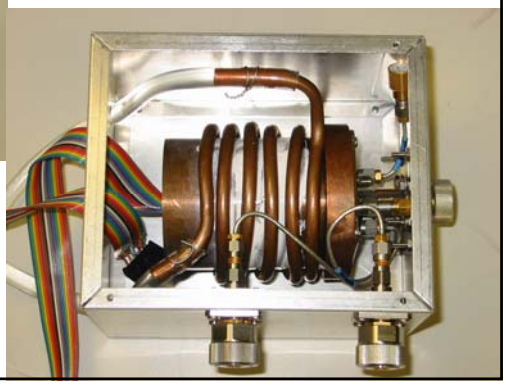


- Present switching unit

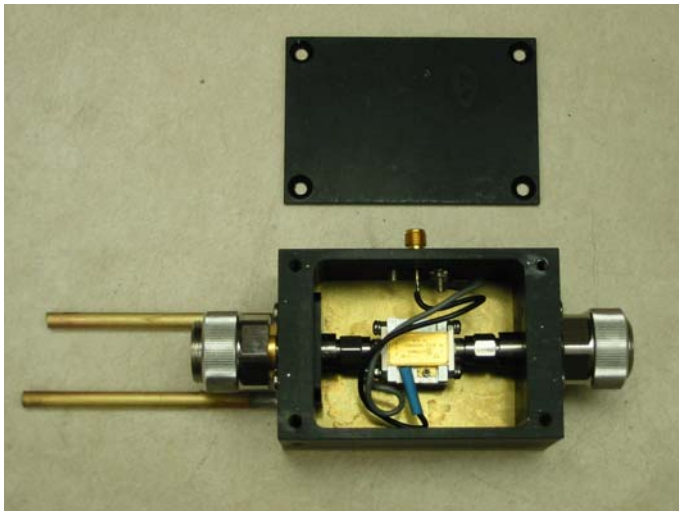


- Developing automated unit (VTU)





- Have boxed amplifiers for circulation in comparisons



Measurement Results

TABLE I
MEASURED VALUES OF THE NOISE PARAMETERS FOR THE AMPLIFIER ALONE.

	8 GHz	9 GHz	10 GHz	11 GHz	12 GHz
X_1(K)	64.5 ± 5.9	67.7 ± 5.5	68.6 ± 5.8	70.4 ± 6.0	80.3 ± 7.0
X_2(K)	110.0 ± 8.4	115.7 ± 7.8	117.6 ± 8.2	124.6 ± 8.4	134.4 ± 10.7
ReX_{12}(K)	9.4 ± 1.5	-7.2 ± 1.2	8.2 ± 1.2	-10.2 ± 1.0	14.3 ± 1.5
ImX_{12}(K)	20.2 ± 1.3	-14.3 ±1.4	10.7 ± 1.5	-14.3 ± 1.6	18.9 ± 2.8
G_0	2031 ± 32	2047 ± 30	1987 ± 30	2121 ± 33	1649 ± 31
T_{min}(K)	112.6 ± 8.4	112.2 ± 7.8	115.1 ± 8.2	123.4 ± 8.4	133.4 ± 10.6
t(K)	128.3 ± 2.6	234.2 ± 3.8	145.8 ± 3.0	223.9 ± 3.9	209.8 ± 4.5
ReΓ_{opt}	-0.172 ± 0.004	0.130 ± 0.004	-0.115 ± 0.005	0.077 ± 0.005	-0.006 ± 0.004
ImΓ_{opt}	0.101 ± 0.008	-0.046 ±. 0.009	-0.003 ± 0.008	-0.004 ± 0.008	-0.069 ± 0.008

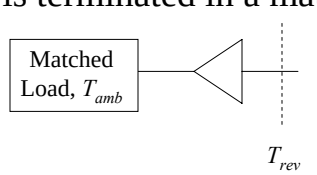
Max(χ^2/DOF) = 0.32

N.b.: in dB,
$G_0 \approx$ 33 dB ± 0.07 dB
$NF \approx$ 1.40 dB ± 0.09 dB

Uncertainties are standard uncertainties (1σ).

Checks & Verification, T_{rev} test

- So what makes us think that we might know what we're doing?
- Have implemented two tests, T_{rev} test and isolator test.
- T_{rev} test: Measure noise temp from input of amplifier, when output is terminated in a matched load.



- Can show that for $\Gamma_L S_{21} S_{12}$ small,

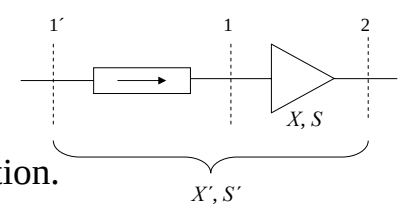
$$T_{rev} \approx \frac{X_1}{\left(1 - |\Gamma_1|^2\right)}$$

- So measure T_{rev}, compare to value predicted from the value of X_1 from the noise-parameter determination. (Use full expression for T_{rev}, not approximation.)
- If working in terms of IEEE parameters, convert to X's to compute T_{rev} & compare to measured value

$$T_{rev} \approx \left[T_{e,\min}\left(\left|S_{11}\right|^2 - 1\right) + \frac{t\left|1 - S_{11}\Gamma_{opt}\right|^2}{\left|1 + \Gamma_{opt}\right|^2} \right]\left(1 - \left|\Gamma_1\right|^2\right)^{-1}$$

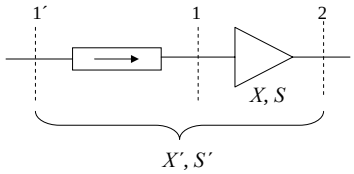
"Isolator" Test

- Connect isolator to amplifier input & measure noise parameters of combination.



- X' parameters can be written in terms of X parameters (amp alone) and the S-parameters of amp and isolator.
- Using Bosma's theorem and standard S-parameter algebra, can show

N.b. Bosma's Theorem: Passive device $\Rightarrow \left\langle \hat{b}_i \hat{b}_j \right\rangle = kT\left(\mathbf{1} - \mathbf{S}\mathbf{S}^{+}\right)_{ij}$



$$X_1' = \left|\frac{S_{12}^I}{1-S_{11}S_{22}^I}\right|^2 X_1 + T_I(A_1 - A_2),$$

$$A_1 = \left\{\left(1-\left|S_{11}^I\right|^2-\left|S_{12}^I\right|^2\right)+\left|\frac{S_{11}S_{12}^I}{1-S_{11}S_{22}^I}\right|^2\left(1-\left|S_{21}^I\right|^2-\left|S_{22}^I\right|^2\right)\right\},$$

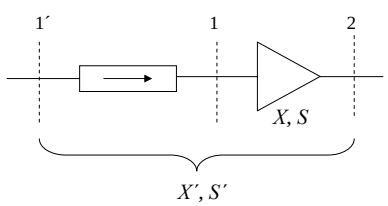
$$A_2 = 2\,\mathrm{Re}\left[\frac{S_{12}^I S_{11}}{(1-S_{11}S_{22}^I)}\left(S_{21}^I S_{11}^{I*} + S_{12}^{I*} S_{22}^I\right)\right],$$

$$X_2' = \frac{1}{\left|S_{21}^I\right|^2}\left\{\left|1-S_{11}S_{22}^I\right|^2 X_2 + \left|S_{22}^I\right|^2 X_1 + 2\,\mathrm{Re}\left[S_{22}^I\left(1-S_{11}S_{22}^I\right)^* X_{12}\right] + T_I\left(1-\left|S_{22}^I\right|^2-\left|S_{21}^I\right|^2\right)\right\},$$

$$X_{12}' = \frac{S_{12}^I\left(1-S_{11}S_{22}^I\right)^*}{S_{21}^{I*}\left(1-S_{11}S_{22}^I\right)} X_{12} + \frac{S_{12}^I S_{22}^{I*}}{S_{21}^{I*}\left(1-S_{11}S_{22}^I\right)} X_1 - T_I A_3,$$

$$A_3 = \left[\left(\frac{S_{21}^{I*}S_{11}^I + S_{12}^I S_{22}^{I*}}{S_{21}^{I*}}\right) - \frac{S_{12}^I S_{11}}{S_{21}^{I*}\left(1-S_{11}S_{22}^I\right)}\left(1-\left|S_{22}^I\right|^2-\left|S_{21}^I\right|^2\right)\right],$$

- Approximate expressions:



$$X_1' \approx T_I,$$

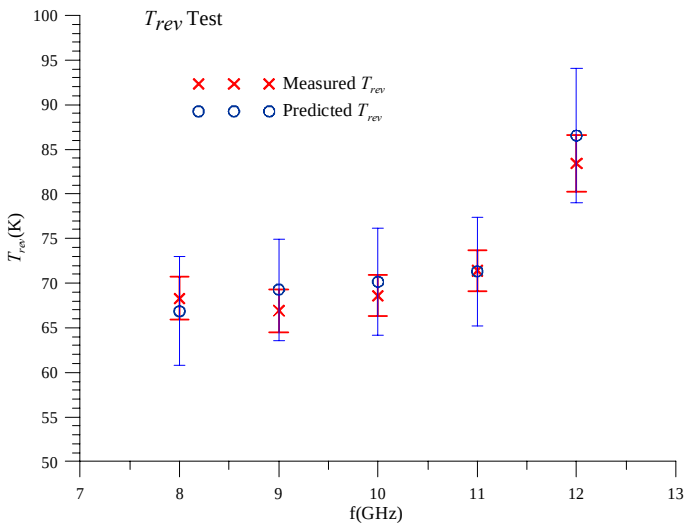
$$X_2' \approx \frac{\left(X_2 + T_I\left(1-\left|S_{21}^I\right|^2\right)\right)}{\left|S_{21}^I\right|^2},$$

$$X_{12}' \approx \frac{S_{12}^I}{S_{21}^{I*}} X_{12} - T_I S_{11}^I,$$

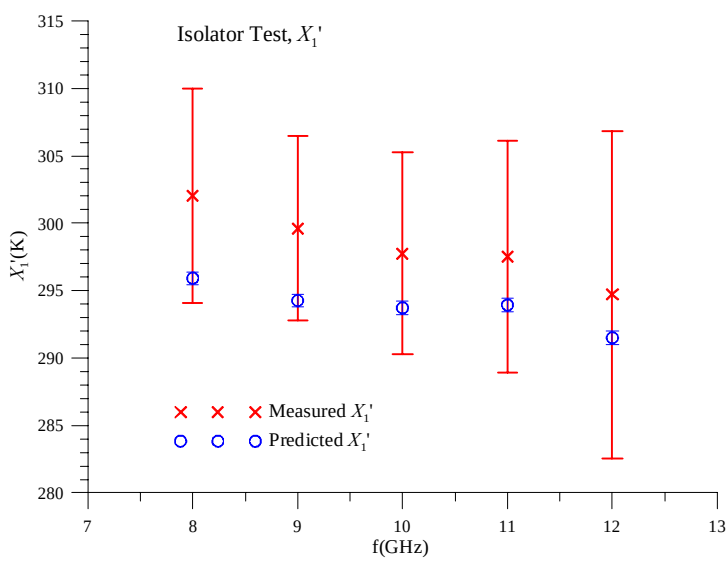
X_{12}' is small and (approximately) independent of amplifier; excellent verification test.

(Could also do with an attenuator in place of the isolator—useful for on wafer.)

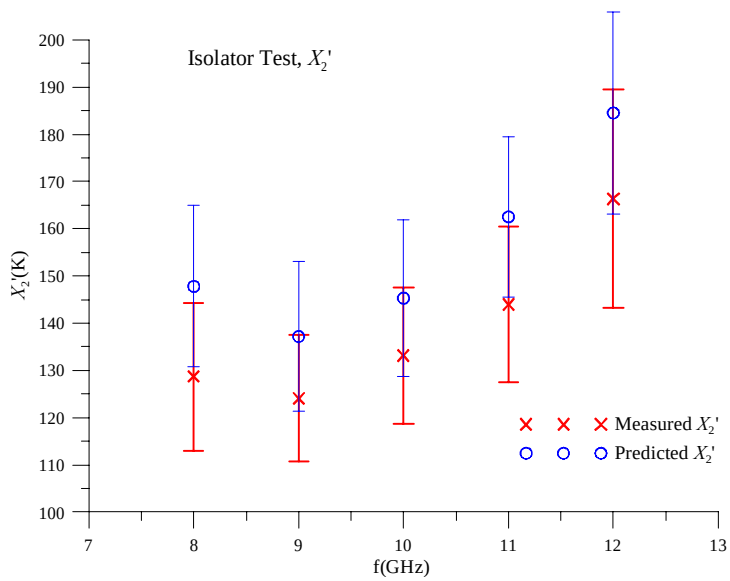
Results of Tests:



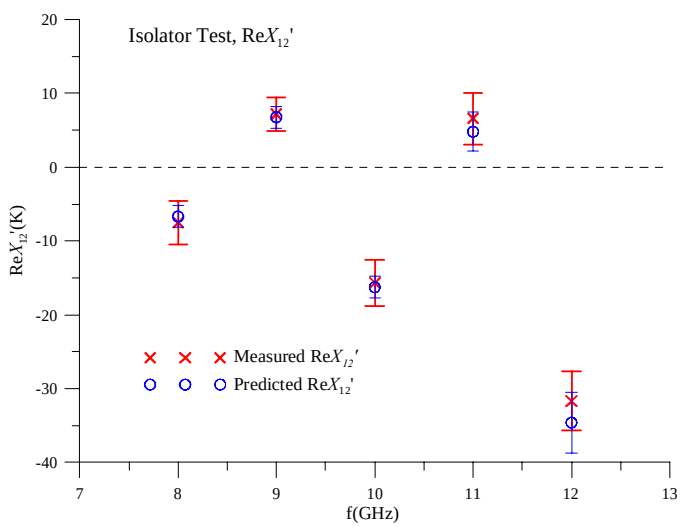
Error bars are standard uncertainties (1σ).



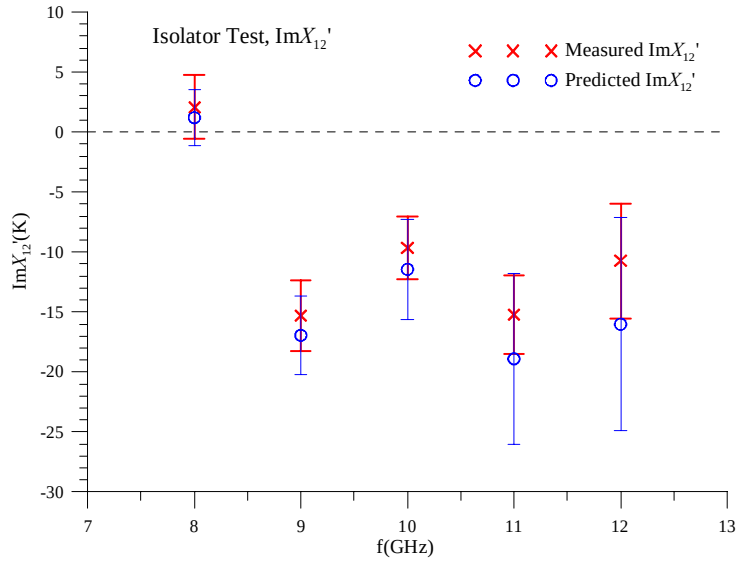
Error bars are standard uncertainties (1σ).



Error bars are standard uncertainties (1σ).



Error bars are standard uncertainties (1σ).



Error bars are standard uncertainties (1σ).

Uncertainties

- Type A (statistical): obtained in the fitting process,

$$u_A(i) = \sqrt{Cov_{ii}}$$

- Type B (other): from Monte Carlo program

- Standard (combined):

$$u_c = \sqrt{u_A^2 + u_B^2}$$

- Uncertainties in IEEE parameters:

$$u_i(IEEE) = \sqrt{V_{ii}(IEEE)}$$

$$V_{ij}(IEEE) = \sum_{i',j'=1}^{5} D_{ii'} D_{jj'} V_{i'j'}(X's)$$

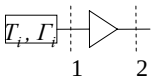
$$D = \begin{pmatrix} \frac{\partial T_{\min}}{\partial X_1} & \frac{\partial T_{\min}}{\partial X_2} & \frac{\partial T_{\min}}{\partial \operatorname{Re} X_{12}} & \frac{\partial T_{\min}}{\partial \operatorname{Im} X_{12}} & 0 \\ \frac{\partial t}{\partial X_1} & \frac{\partial t}{\partial X_2} & \frac{\partial t}{\partial \operatorname{Re} X_{12}} & \frac{\partial t}{\partial \operatorname{Im} X_{12}} & 0 \\ \frac{\partial \operatorname{Re} \Gamma_{opt}}{\partial X_1} & \ldots & \ldots & \ldots & 0 \\ \frac{\partial \operatorname{Im} \Gamma_{opt}}{\partial X_2} & \ldots & \ldots & \ldots & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

- Monte Carlo for type-B uncertainties:
  - input uncertainties in reflection coefficients, noise temperature of non-ambient source, ambient temperature, measurement of output noise temperature (or power), correlations, ...
  - generate simulated measurement values for everything; analyze/fit as if it were real data; do it again (and again and again and …): simulate, analyze, repeat.
  - $u_B(y) = \sqrt{Var(y) + (y_{true} - \bar{y})^2}$

- Some general approximate features:
  - Uncerts in G and $T_{\min}$ (& $F_{\min}$) are dominated by uncert in T_h. 0.1 dB uncert in $T_h \rightarrow \sim$ 0.1 dB uncert in G_0 and $F_{\min}$.
  - Uncerts in Γ_{opt} are dominated by uncerts in Γ_G's. Uncert in Re or Im Γ_{opt} is ~ 3 or 4× uncert in Re or Im Γ_G (for 13 terminations).
  - t is sensitive to just about everything.
  - T_{amb} is not a major factor, because it is known much better than T_h. Note, however, that it could affect T_h or the amplifier properties.
- Have also used simulation to compare uncertainties for different measurement strategies.

IV. On-Wafer Noise Parameters

- Working with IBM & RFMD
- Complications on wafer:
  - characterizing & correcting for probes
  - loss through probes → can't get too close to $|\Gamma| = 1$ for input terminations
- Device properties problematic:
  - poorly matched: $|S_{11}|$, $|S_{22}|$, $|\Gamma_{opt}| > 0.5 \Rightarrow$possible power transfer problems; non-physical results more common
  - $|S_{12}S_{21}|$ not very small → $|\Gamma_2| > 1$ in some cases
  - very low noise, $T_e \sim 10 - 30$ K
  - not necessarily stable



Bounds & Unphysical Results

- Two most commonly violated bounds are

$$X_1 = T_{e,\min}\left(\left|S_{11}\right|^2 - 1\right) + \frac{t\left|1 - S_{11}\Gamma_{opt}\right|^2}{\left|1 + \Gamma_{opt}\right|^2} \geq 0$$

$$T_{e,\min} = \frac{X_2\left(1 - \left|S_{11}\Gamma_{opt}\right|^2\right) - \left|\Gamma_{opt}\right|^2\left[X_1 - 2\,\mathrm{Re}\left(S_{11}^* X_{12}\right)\right]}{\left(1 + \left|\Gamma_{opt}\right|^2\right)} \geq 0$$

- Have done simulations to see how often one obtains unphysical results
- Results depend on everything, of course, but representative results are:

Input Uncertainties (correlated & uncorrelated)

	Good	Less Good
Γ(<0.5), cor	.0021	.003
Γ(<0.5), uncor	.0021	.0021
Γ(>0.5), cor	.0035	.004
Γ(>0.5), uncor	.0035	.0035
S_{21}, cor	.007	.007
S_{21}, uncor	.007	.007
Connector repeatability	.001	.001
$T_{ambient}$, cor	0.5 K	0.5 K
$T_{ambient}$, uncor	0.0	0.5 K
T(not near ambient), cor	0.7 %	1.0 %
T(not near ambient), uncor	0.7 %	0.7 %

Percent unphysical results

	Good Uncerts	Less Good
Device 1	0.9 %	1.4 %
Device 2	9.4 %	11.5%

In Progress with IBM & RFMD

- Measure S-parameters & noise parameters of MOSFETs with 0.13 μm gate length; inter-compare.
- Evaluate our uncertainties, and theirs (?).
- Incorporate measurement of T_{rev} on wafer.
- Apply checks on wafer:
  - compare (attenuator + transistor) to (transistor)
  - noise parameters of mismatched attenuator
  - T_{rev}

V. Next

- Continue on-wafer work with IBM & RFMD
- Amplifier noise-parameter comparison with NPL
- Measurements on cryogenic amplifier with THz Project
- Considerable amount of data to analyze, document

- Automate measurements & data handling
- Improve measurements—cold input source, better choice of input terminations, …
- Round-robin with industry?