

S. R. Stein  
 Frequency and Time Standards Section  
 National Bureau of Standards  
 Boulder, Colorado 80302  
 (303) 499-1000, extension 3224

and

H. van de Stadt\*  
 Joint Institute for Laboratory Astrophysics  
 National Bureau of Standards and University of Colorado  
 Boulder, Colorado 80309

### Summary

An electronic tuning technique for far-infrared (FIR) lasers has been developed which makes it possible both to frequency modulate a laser at a very high rate and to phase-lock it to a reference oscillator. Tuning is achieved by applying an electronic (Stark) field to the molecules in an optically pumped FIR laser. Since such lasers operate on rotational-vibrational transitions of molecules, they exhibit either first- or second-order Stark effects which result in an effective broadening of the molecular transition. This introduces a change in the cavity pulling, which is present in any laser in which the cavity resonant frequency is offset from the molecular transition.

Using a  $\text{CH}_3\text{F}$  laser at a frequency of 604 GHz (496  $\mu\text{m}$ ), we have observed (by direct measurements with a metal-insulator-metal diode) a maximum static electronic tuning of 500 kHz with a maximum sensitivity of 5 kHz/(V/cm). The tuning rate should be limited only by the lifetime of the photons in the laser cavity. We have observed frequency modulation rates of 300 kHz and a modulation index greater than one has been obtained at a modulation rate of 50 kHz.

We report the ability to phase lock such a tunable FIR laser to a reference laser at an offset frequency of the order of 500 kHz. This was achieved by using the Stark-tuning method to correct for fast frequency fluctuations, up to approximately 20 kHz, and a piezo-electric tuner (PZT) to slowly correct for changes in the laser cavity length.

The tuning and phase-lock methods reported here are needed for several important applications, for example: infrared frequency synthesis, generation of time from infrared (and higher) frequency standards, directional and synthesized infrared communications, and FM LIDAR.

### Experimental Description

A schematic diagram of the experimental setup is shown in Figure 1. A  $\text{CO}_2$  laser and amplifying tube provide the pump beams, at a wavelength of approximately 10  $\mu\text{m}$ , necessary to excite a particular vibrational level of the molecules in FIR lasers I and II. One laser, FIR-I, serves as a reference laser. A metal-insulator-metal diode with a 50  $\mu\text{m}$  diameter tungsten whisker and a polished nickel post is used for heterodyning the two FIR frequencies. More details about the setup can be found in Ref. 1.

The FIR-II laser consists of a glass tube, 1.04 m long and 32 mm internal diameter. Two lengths of copper tape are attached over the total length of the tube to serve as the internal Stark electrodes. Both ends of the tube are closed by flat, metal mirrors, of which one is movable in two different ways: A PZT is used for fine movements (4  $\mu\text{m}$  maximum), while a micrometer drive provides the coarse motion.

A block diagram of the phase-lock loop system is shown in Figure 2. The two FIR lasers produce a beat frequency on the order of several hundred kilohertz in the metal-insulator-metal diode detector. After amplification and filtering this beat note is fed into a double-balanced mixer, where it is compared to the frequency of a synthesizer. The phase-lock loop tunes the FIR-II laser so that the heterodyne signal between the two lasers and the reference signal from the synthesizer are in approximate phase quadrature. This is accomplished in two different ways: Fast tuning is achieved by amplifying and filtering the detected phase-difference signal and then feeding it to the Stark electrodes. The attack time in this feed-back loop is approximately 10  $\mu\text{s}$ . Slow tuning, over a larger range, is mainly necessary to compensate for length changes of the FIR-II laser cavity due to temperature changes. This tuning is achieved by feeding the detected phase-difference signal through an integrating amplifier and applying it to the PZT-tuning element of the FIR-II laser. Unwanted interactions between the two feedback loops are not present because of the large difference in speed ( $10^4$ ) of the two mechanisms. The present system stays in a phase-locked condition for about 15 minutes. This time is mainly limited by the frequency instability of the  $\text{CO}_2$ -pump laser and the mechanical instability of the diode, rather than by the dynamic range of the phase-lock loop itself.

### Measurements

All measurements are preceded by mechanically tuning the FIR-II laser to a dominant longitudinal mode. In Figure 3, we show the output power variation as a function of cavity length for the Stark tunable laser. The two main maxima occur at a cavity length difference of 248  $\mu\text{m}$ , half the wavelength of the 496  $\mu\text{m}$  transition in  $\text{CH}_3\text{F}$ . The relatively clean mode spectrum of this laser avoids the otherwise annoying troubles due to mode interference.

The next step is to first detune the cavity slightly from the transition line center and then apply a dc voltage of about 400 V to the Stark electrodes. This is done while the FIR-I laser remains tuned to its line center, allowing one to monitor the frequency changes of FIR-II by direct heterodyning in a frequency region of the order of 500 kHz. The initial cavity detuning produces a frequency shift of

\*Visiting Fellow 1976-77, on leave from Astronomical Institute, Utrecht, The Netherlands.

300 to 600 kHz in the FIR-II laser, while the additional amount of electronic tuning, which is proportional to the initial cavity detuning, has a maximum sensitivity of 5 kHz/(V/cm). Associated with the frequency change there may be an amplitude change with a relative magnitude of 0.01/(V/cm).<sup>1</sup>

The application of an alternating field, superimposed on the dc electric field, produces both amplitude and frequency modulation sidebands. The ratio of AM sidebands to the first FM sidebands is expected to be  $2 \times 10^{-6} f_m$ , where  $f_m$  is the modulation frequency in Hz, as calculated from the sensitivities quoted in the preceding paragraph.<sup>1</sup> It has been verified qualitatively that the AM and FM sidebands have indeed equal strength at  $f_m = 500$  kHz.

Figure 4 shows a typical frequency spectrum of a frequency-modulated, Stark-tuned FIR laser. It was taken using a spectrum analyzer connected to the amplified heterodyne signal from the diode. Since only the FIR-II laser is modulated, it represents the power density of this laser as a function of frequency. This particular spectrum was taken while the laser was modulated at 50 kHz. The relative magnitude of the sidebands is typical for frequency modulation with a deviation (half the peak-to-peak frequency shift) equal to 75 kHz. The AM sidebands in this case are ten times less powerful than the FM sidebands, in agreement with our earlier statement and indicating that we have a nearly pure frequency modulated spectrum.

Observation of the beat note between the two FIR lasers indicates that the 3 dB line width of the unlocked lasers is a few kHz. A typical spectrum is shown in Figure 5a. In order to phase lock such oscillators it is necessary that all the amplifiers and filters in the loop be broadband compared to the free-running line width. When the phase-lock loop is operating, the line width of the beat note decreases dramatically as can be seen in Figure 5b. If a stable reference were substituted for the FIR-I laser, then this change would represent a proportional increase in stability of the tunable laser.

The most important parameter describing the phase-lock loop is its attack time, i.e., the inverse of the unity gain angular frequency. It can be measured by phase modulating the synthesizer which generates the frequency offset between the two lasers. When the frequency of this phase modulation is small compared to the unity gain frequency, coherent modulation sidebands of constant amplitude are impressed on the tunable laser. However, when the modulation frequency is large compared to the unity gain frequency, the tunable laser can't follow the phase modulation of the reference. We observe this phenomenon in both the spectrum of the beat between the two lasers and the spectrum of the control voltage to the tunable laser. Our best estimate of the unity gain frequency is 18 kHz which corresponds to an attack time of approximately 10  $\mu$ s.

In order to analyze the operation of the servo system it is useful to examine the spectrum of the feedback voltage to the Stark tuning electrodes. An example of such a spectrum is shown in Figure 6. The primary value of this measurement is to detect instabilities in the loop which show up as peaks in the spectrum. The loop is observed to be stable, but begins to oscillate when the gain is increased by 10 dB. It is also possible to use the spectrum of the feedback voltage to determine  $S_\phi(f)$  for the pair of oscillators. This is discussed in more detail in the next section.

When the Stark tuner is operating alone, the dynamic range of the loop is very limited. For example,

a frequency change of the reference oscillator in excess of 50 kHz causes the loop to come out of lock. This dynamic range is insufficient to control the slow frequency changes in the FIR lasers which we surmise are due to temperature changes. The inclusion of the PZT tuner, which can move one end mirror  $\pm 2$   $\mu$ m, results in an order of magnitude improvement in the dynamic range.

### Interpretation

We selected the 496  $\mu$ m (604 GHz) laser transition in CH<sub>3</sub>F for this study because it is a relatively strong line and the Stark effect in CH<sub>3</sub>F is linear and well understood. The CO<sub>2</sub> laser selectively pumps the molecules first to an excited vibrational level and the resulting population inversion induces the 496  $\mu$ m rotational transition to lase. An electric field splits the rotational transition into several components, each with a known frequency shift proportional to the field strength. From the known line width of the individual components one can show<sup>1,2</sup> that the net effect of the Stark field is an overall broadening of the line of approximately 30 kHz/(V/cm).

The tuning of the laser frequency is produced by this broadening via the effect of "cavity pulling."<sup>3</sup> The actual laser frequency,  $v$ , is determined by:

$$\frac{v-v_m}{\Delta v_m} = \frac{v_c-v}{\Delta v_c},$$

where  $v_c$  is the resonant frequency of the empty laser cavity and  $\Delta v_c$  its full width,  $v_m$  is the frequency of the molecular transition and  $\Delta v_m$  its full width. It is clear from this equation that an offset between  $v_c$  and  $v_m$  produces a static frequency shift of the laser frequency towards the cavity line center. The amount of shift depends on the cavity offset itself, as well as the magnitudes of  $\Delta v_c$  and  $\Delta v_m$ . The very last dependence is used in our experiments.

Differentiation of the pulling equation with respect to the applied field  $E$  yields:

$$\frac{dv}{dE} = \frac{\Delta v_c(v_c - v_m)}{(\Delta v_m + \Delta v_c)^2} \frac{d}{dE} (\Delta v_m).$$

The shift is thus proportional to the initial offset of the cavity from line center and the proportionality factor can be calculated from known quantities. We find in our case a maximum value of 5 kHz/(V/cm), in good agreement with measured values.

The operation of the phase-lock loop, described earlier, is best understood in terms of the open-loop transfer function  $G_{eq}(j\omega)$ .<sup>4</sup> Let us call the open-loop phase fluctuations of the two lasers  $\phi_I$  and  $\phi_{II}$ , respectively. Then the closed-loop phase fluctuations between the two lasers,  $\Delta\phi$ , is

$$\Delta\phi = \frac{\phi_{II} - \phi_I}{1 + G_{eq}},$$

where the frequency synthesizer is assumed to have negligible phase fluctuations. The open-loop transfer function is the product of the tuning rate of the FIR-II laser, the sensitivities of the MIM diode and the double balanced mixer, the gain of amplifiers  $G_1$  and  $G_2$ , and the low pass transfer functions  $F_1$ ,  $F_2$  and  $F_3$ . The asymptotic form is shown in Figure 7. The break point at  $f_2$  is due to the low pass filter ( $F_2$ ) which decreases the feedback of the second harmonic of the offset frequency

between the two lasers. The break point at  $f_3$  is due to the finite band width ( $F_3$ ) of the amplifiers. The phase shift caused by these two low pass filters currently limits the maximum unity gain frequency to about 20 kHz. The break at  $f_4$  is due to the PZT tuner and has negligible influence on the second-to-second performance of the loop.

The voltage,  $V$ , measured at the spectrum analyzer is easily related to the open-loop phase fluctuations between the two lasers. Since the spectrum analyzer follows filter  $F_2$ ,

$$V = \frac{K(\phi_{II} - \phi_I)}{(1+jf/f_2)(1+G_{eq})},$$

where  $K$  is a constant. Solution of this equation indicates that the slope of the voltage spectrum should change by  $f^2$  at a frequency near the unity gain frequency. The break does not occur exactly at the unity gain frequency mainly due to the fact that the poles at  $f_2$  and  $f_3$  are not far removed. Examination of the spectrum in Figure 6 shows just such a slope change at 18 kHz. It also shows a slope change at 4 kHz, below which the voltage spectrum is constant. This behavior is consistent with the interpretation of 4 kHz as the fast linewidth (approximately equal to the 3 dB linewidth) of the unstabilized lasers.<sup>5</sup>

Since the time constants of the various filters in this phase-lock loop are not widely spaced,  $S_\phi(f)$ , the spectral density of phase fluctuations between the two lasers, must be calculated on a point-by-point basis. We find that  $S_\phi(30 \text{ kHz}) \approx -90 \text{ dB}$  relative to 1 rad<sup>2</sup>. This noise level is comparable to a Gunn-effect oscillator referred to its operating frequency of 10 GHz.

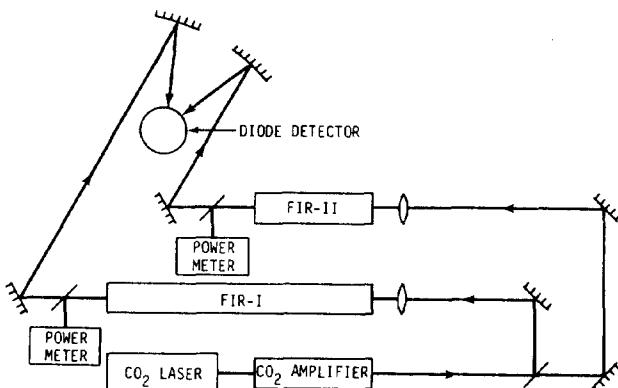


FIGURE 1. Schematic diagram of the system for heterodyning two far infrared (FIR) lasers.

### Conclusions

In the near future, we expect the expansion of the fields of time and frequency and telecommunications into the infrared to accelerate significantly. It is likely that the far-infrared laser will be a workhorse oscillator for that frequency range. To promote this possibility, it is necessary to develop a variety of modulation and phase-locking techniques such as are available in the microwave region. The Stark frequency modulation and phase-lock technique which is described here is a step in this direction. When combined with suitable low noise microwave sources and efficient multipliers it will permit frequency synthesis over a large portion of the far-infrared wavelength region.

### References

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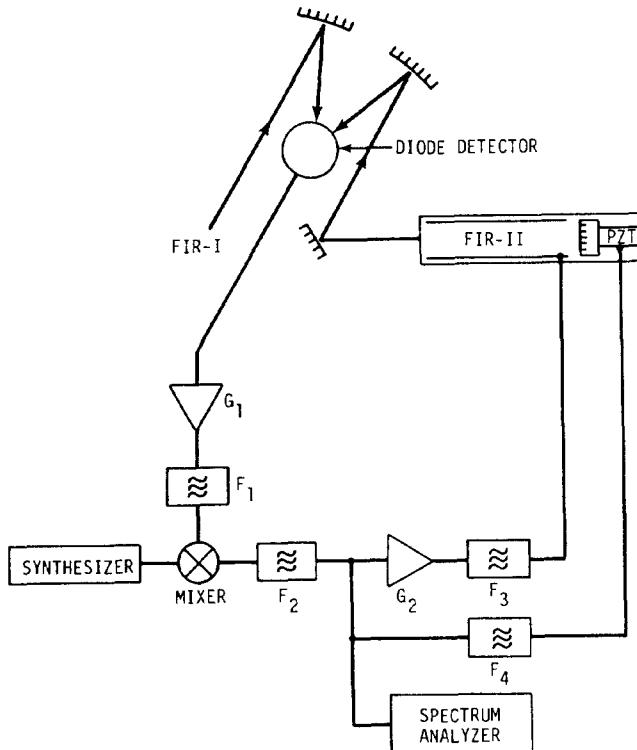


FIGURE 2. Block diagram of the phase-lock loop system.

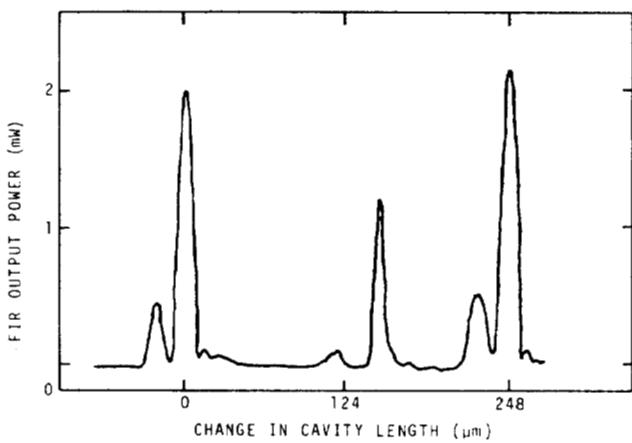


FIGURE 3. Output power of the FIR laser as a function of cavity length. In addition to the main cavity modes, there are visible a small number of higher order modes.

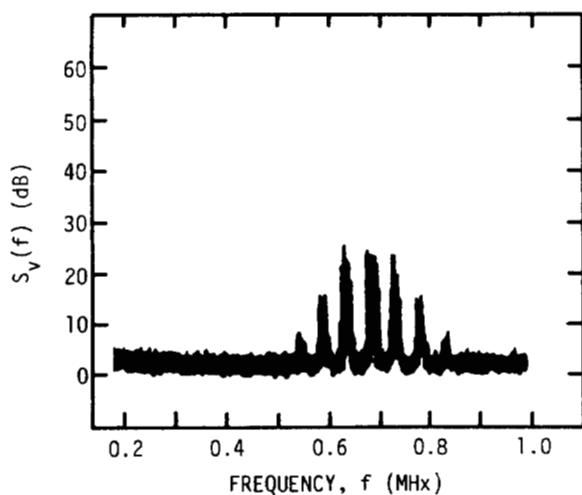
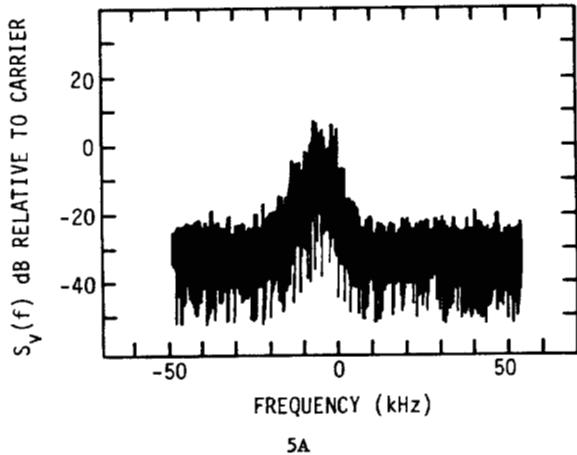
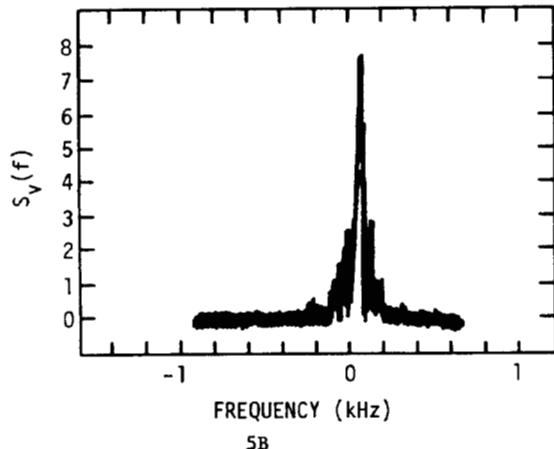


FIGURE 4. Frequency spectrum of a Stark-tuned FIR laser with a modulation frequency of 50 kHz.



5A



5B

Fig. 5. a) Beat frequency spectrum of two unlocked FIR lasers.  
b) Beat frequency spectrum of two phase-locked FIR lasers.

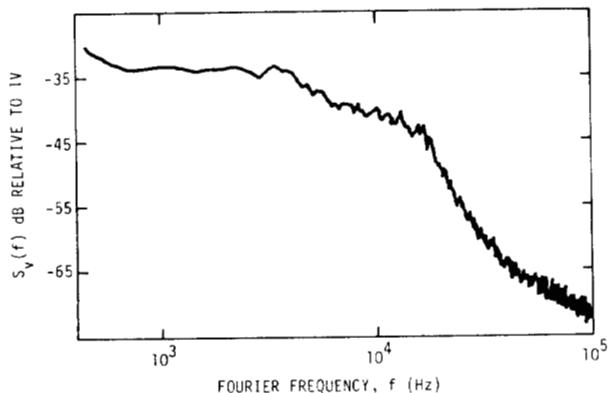


FIGURE 6. Frequency spectrum of the feedback voltage in the closed-loop condition.

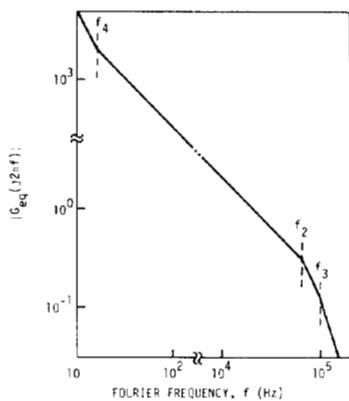


FIGURE 7. Asymptotic form of the magnitude of the open-loop transfer function.