Microwave Power Measurements

NIST/ARFTG Measurements Short Course
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Tom Crowley
NIST, MC 818.01
325 Broadway
Boulder, CO 80305
crowley@boulder.nist.gov

Outline

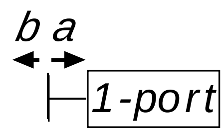
1. What is measured (and why?)
   - Why power?
   - CW & Pulsed power parameters
2. How do we measure power?
   - Traceability
   - Sensors: Bolometer
     - Thermocouple
     - Diode
   - Power Meters
3. Measurement Examples and Techniques
   - Calibration Factor
   - Mismatch Factor
   - Uncertainty
4. Future

An emphasis will be placed on the basic measurement concepts and technologies along with some important considerations for those making measurements (mismatch factor). Primary standards will be covered briefly. Less common sensor types will not be covered.

My background:
I am the project leader for microwave power at NIST. I have been working with the primary standards (calorimeters) and transfer standards (bolometers) since September, 2000. Our transfer standards are mostly modified versions of commercial sensors and are controlled with NIST Type IV power meters.

Power: What is measured?

Signal level ("Power" does not imply high power)
Fundamental
(Comparable to voltage at DC and low f)
Defined as energy/time (averaged over time)
Primary standards - measure energy as heat



a, *b* are complex amplitudes used by VNA ($S_{11} = b/a$)
comparable to voltage, current

$P_{inc} = |a|^2$; $P_{ref} = |b|^2$; $P_{net} = |a|^2 - |b|^2$

RF power is one of the most critical microwave measurements. It affects the cost and performance of most microwave devices. Power levels that are too low result in reduced performance. Power levels that are too high waste resources (e.g. battery life in a cell phone), increase costs, and reduce operational flexibility.

Power refers to signal level. Primary standards are typically calibrated at 1-10 mW.

Power = Energy/time: The energy is stored in the electric and magnetic fields. In many power sensors, this energy is converted to heat and measured. The averaging time depends on the application. It is at least an RF period.

Relationship between a, b, 'voltage', 'current', and power
As outlined in reference [2}, you can define a voltage and current with:

$$v = a + b$$
$$i = (a - b)/Z_0$$

Inverting these equations yields:

$$a = (v + iZ_0) / 2$$
$$b = (v - iZ_0) / 2$$

where Z_0 is an arbitrary reference impedance (not necessarily the characteristic impedance). If $Z_0 = 1$ is used, then the net power is given by $P = \langle i^*v \rangle = |a|^2 - |b|^2$ where < > indicates a time average.

Why power?

Power vs voltage/current or E/H

	Advantages	Disadvantages
Power	Not f(z) Straightforward measurement What user wants Energy must be conserved	Traceability to fundamentals
Voltage	Tie to low frequency measurements VNA measurements	$V(z)$ How to define voltage Harder to measure
Electric or Magnetic Field	Traceability to fundamental parameters*	$\boldsymbol{E}(x,y,z)$ and $\boldsymbol{H}(x,y,z)$

*NIST has 1 research effort measuring ***E*** and another measuring ***H***

When an RF signal propagates between two devices, energy must be conserved. If two devices with different characteristic impedances interact, the power must be conserved, but there is no corresponding requirement for the voltage. (Things can be predicted and corrected for, but generally it is easier to just use power).

Traceability for RF power is through DC power and therefore DC voltage and resistance. The largest uncertainty is due to differences in the RF and DC power dissipation.

Josephson junction experiments with outputs in the MHz range have been done.

Standing wave patterns produce a variation in the voltage and current with period $\lambda/2$. It can therefore be critical where you measure the voltage. In contrast, the forward and reflected power do not vary with position.

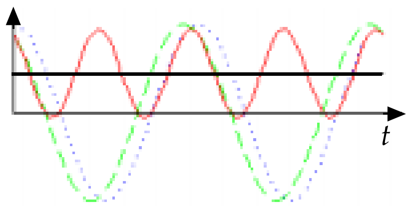
A voltage associated with a time varying electric field depends on the path. In some cases, there is an obvious natural choice to define voltage, but in others it is not clear.

The electro-optic sampling system measures the RF electric field and traces back to a DC voltage measurement.

A quantum-based Rabi oscillation measurement has measured the RF magnetic field by the flipping of cesium atoms between two states.

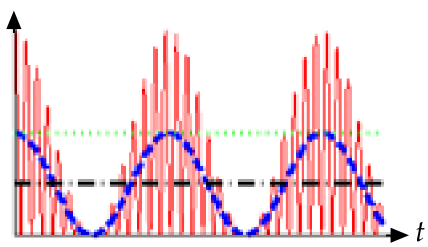
What is measured - Power Definitions

CW RF signal



Avg over carrier period - “Envelope Power”

Two tone signal



$p(t) = V(t)\,I(t)$

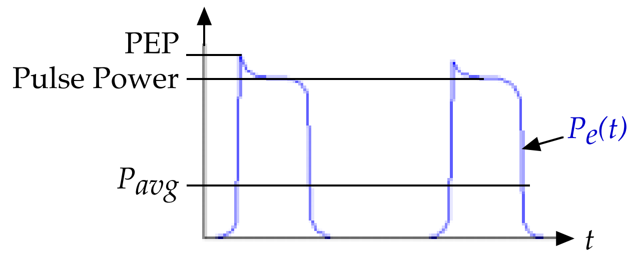
$$P_e(t) = \frac{1}{T}\int_{t-T/2}^{t+T/2} p(t')dt'$$

An “RF power” measurement refers to an average over multiple cycles of the carrier frequency.
None of the detectors described in this talk have an output corresponding to p(t). Some have outputs corresponding to $P_e(t)$, P_{avg}, or other parameters described.

Envelope power - power averaged over the period of the carrier
Has same shape as the envelope of *p(t)*
Average power - power averaged over the modulation period
Peak envelope power - sometimes referred to as peak power

What is measured - Power Definitions

Pulsed signal (e.g. TDMA)



Pulse power is average envelope power during the pulse.

Adjust the time window being studied for Pulse Power (and other) measurements.

Crest factor - PEP / P_{avg}

Video bandwidth / Modulation Bandwidth

Most sensors - wide range of carrier frequencies (up to 50 GHz range). (*Not* the video bandwidth).

Video bandwidth - frequency range of the signal envelope or modulation that can be measured. You need a sensor with video bandwidth greater than the modulation bandwidth of the signal effects you are studying.

A pulse power measurement can be performed between two time windows on some power meters. You would typically use the half power points on the rise and fall to denote the time window.

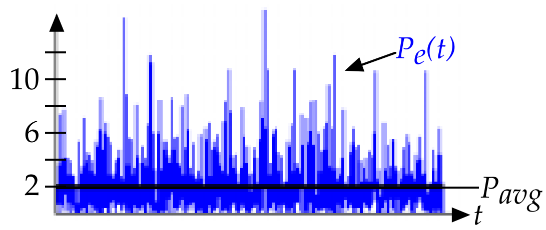
The carrier signal is *p(t)* from previous slide.

The modulation bandwidth of importance depends on what you desire to look at. For example, you need a wider bandwidth to be able to detect the initial overshoot (responsible for the PEP signal) than you do for the pulse power measurement. In the case of a two tone measurement, the bandwidth is the separation between the two tones.

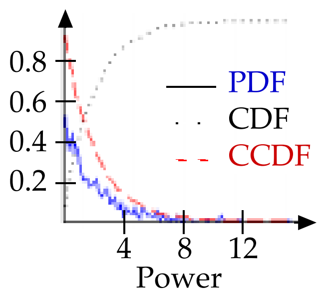
There can be a tradeoff between video bandwidth and sensitivity since a narrower bandwidth allows more noise to be filtered out.

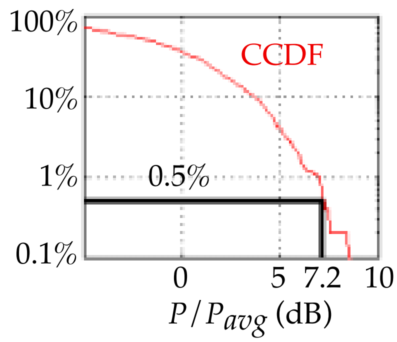
Only diode sensors are fast enough to respond to most modulation effects of interest.

What is measured (and why?)
Power Definitions - CDMA Signal



Envelope power can be 10x P_{avg} and is highly variable.
PEP doesn't tell you everything you need to know.





PDF - Probability Distribution Function

PDF(x) dx = probability that power is between x and x+dx

CCDF(x) - Probability that power level is greater than x

Top figure - 1000 points from a Gaussian-like distribution.

PDF is normalized so that $\int_0^\infty PDF(x)dx = 1$

CCDF - Complementary Cumulative Distribution Function

$CCDF(x) = \int_x^\infty PDF(x)dx$ - in other words, you sum up all the probabilities that are greater than x which gives you the probability that the power is greater than x. This can be used to tell you how often an amplifier will saturate. By comparing different stages you can tell if you have compression, etc. In the figure on the right, the CCDF tells you that 0.5% of the time, the power level will be more than 7.2 dB higher than the average power.

CDF(x) - Cumulative Distribution Function
Probability that power will be less than x
$CDF(x) = \int_0^x PDF(x)dx$
CCDF(x) = 1 - CDF(x)

The shapes of PDF and CCDF are the same for this example, but that is not in general true. (Its a consequence of the mathematical model I used to generate the top graph).

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   - ***Sensors: Bolometer***
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     - ***Diode***
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How do we measure power?
Traceability

RF power - measured in terms of DC power
- Look for levels that produce same heat
- fundamental traceability is through DC V & R
- *(RF / DC difference - efficiencies & correction factors)*

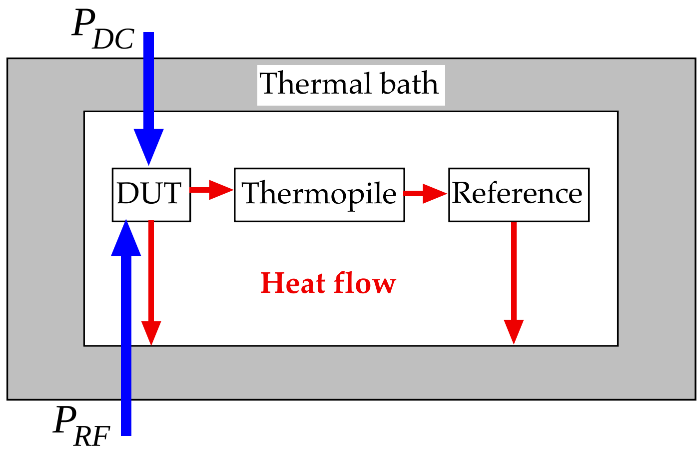
Primary Standards
- National labs - calorimeter
- temperature difference - RF and DC power measured with thermocouple

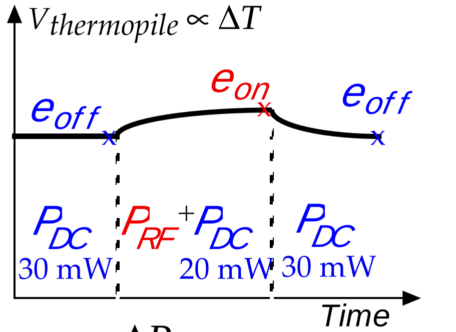
Transfer Standards
- Some measure in terms of DC power, some don’t

The RF/DC substitution isn’t perfect mostly because the heat tends to be deposited in slightly different regions. An important parameter in an RF/DC substitution measurement is the ratio between signal levels for an equal amount of RF or DC power. For the calorimeter measurement, this ratio is characterized by a correction factor, g. For transfer standards that use RF/DC substitution, this ratio is called the effective efficiency, η_e.

NIST’s transfer standards are characterized by operating them in a calorimeter. (Also true for customer mounts that are NIST CN mounts). These transfer standards are then used to calibrate most customer mounts using 6 ports or direct comparison systems.

Calorimeter





$$P_{RF} = \frac{\Delta P_{DC}}{g} \frac{e_{on}}{e_{off}} >\approx 10mW$$

DC Substitution - Measure power in terms of heat.
Measure heat flow from DUT (bolometer) with RF and DC.
DUT and reference are thermally isolated from bath.
g is a correction factor.

NIST 2.4 mm calorimeter

I will use the terms calorimeter and microcalorimeter interchangeably.

For the purist, these are not true calorimeters. A true calorimeter measures energy, not power.

NIST calorimeters have DC power supplied during both the on and off periods.
Typically, $P_{DC,off}$ ~ 30 mW, $P_{DC,on}$ ~ 20 mW, and P_{RF} ~ 10 mW.

I won't discuss commercial calorimeters because they aren't as widely used as bolometers, thermoelectrics, and diodes. At present, there are at least 2 models for high frequency waveguide bands. Historically, there have also been high power applications that used calorimeters. Commercial calorimeters operate on the principle of measuring temperature changes, but are significantly different from what is presented here. They are used to directly measure power rather than evaluate a transfer standard. Above 110 GHz, this is the only option for making an absolute power measurement that I am aware of.

How do we measure power? - Sensor Technologies (Transfer Standards)

1. Bolometer - Thermistor & Thin Film
2. Thermoelectric
3. Diode

Emphasis will be on sensors used with power meters. Simple diode detectors used without power meters won't be specifically discussed although they share some properties with the more complicated sensors.

How do we measure power? - Sensor Technologies (Transfer Standards)

Bolometer -> Thermoelectric -> Diode
less accurate
better sensitivity
better dynamic range
faster

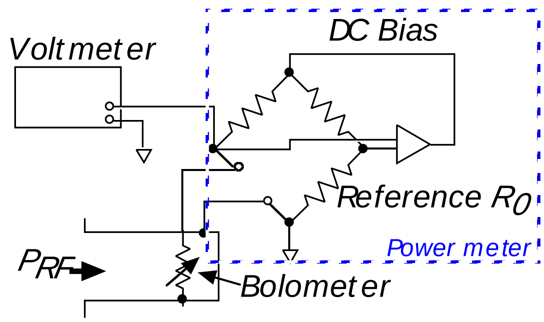
Discuss only terminating sensors

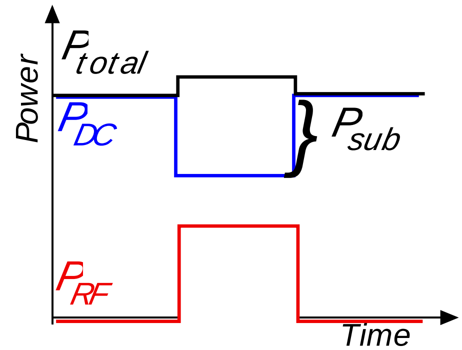
The dynamic range of a thermoelectric detector is comparable to the linear dynamic range of a diode.

I will only discuss sensors that terminate the signal. Feedthrough or directional power sensors are also available. These measure the power without terminating the signal. At least some of these sensors use the same type of sensors as terminating sensors (e.g. diodes) in combination with a directional coupler.

How do we measure power? - Bolometer Sensor

Bolometer - temperature sensitive resistor
Types - ***Thermistors***, Barretters, Thin film
Heat with DC, RF in a bridge circuit or NIST Type IV - DVM measures voltage





Effective Efficiency $\eta_e = P_{sub}/P_{RF,net}$; $P_{sub} = (V_{DC,off}^2 - V_{DC,on}^2)/R_o$

Calibration Factor $CF = P_{sub}/P_{RF,inc} = \eta_e(1-|\Gamma|^2)$

The NIST Type IV power meter is not actually a bridge circuit, although it functions very similarly to one. (reference 8).

Effective efficiency is measured in a calorimeter or by comparison with a transfer standard.

$V_{DC,off}$ and $V_{DC,on}$ are the DC voltages measured by the DVM when the RF is off and on respectively.

How do we measure power? - Bolometer Sensor

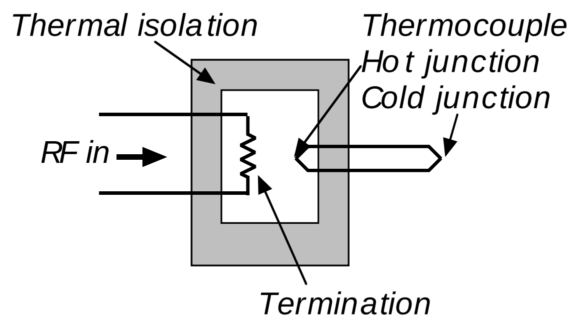
- Accurate
  - NIST transfer standards are bolometers
  - NIST uncertainty (k=2) - 0.0024 -> 0.025
- Slow
- Limited dynamic range
  - -20 dBm -> +10 dBm
- Very good linearity (< 10 mW)
- SWR tends to be high

Uses

- Metrology
- CW signals

In dual element thermistors used in some coaxial sensors, an imbalance in the two thermistors results in nonlinearity effects above 10 mW.

How do we measure power? - Thermocouple Sensor



RF heats termination, $\Delta T \propto P_{abs}$

Thermocouple - 2 metals, temp difference at junctions induces voltage, $V \propto \Delta T$, P_{abs}

Need to amplify V (in sensor and power meter)

Measure power through the amount of heat produced.
Therefore, it is a thermal sensor (as are bolometers).
However, there is no comparison with DC thermal effects, so it is not a DC substitution measurement.

Require power meter for operation to amplify voltage.

RF at different frequencies will heat the termination resistor and input section differently. Thus the thermocouple voltage varies with frequency. Characterize this with “calibration factor”.

How do we measure power? - Thermocouple Sensor

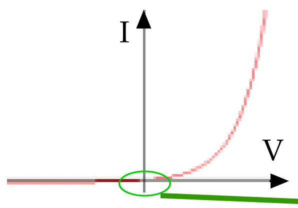
Good Dynamic Range 50 dB
-30 dBm -> +20 dBm (typical, also up to +45 dBm)
Good "Linearity"; $V_{out} \propto P_{abs}$

Uses
Average power for any signal modulation
CW

Higher power levels (+45 dBm) can be obtained by attenuating input, get similar dynamic range.

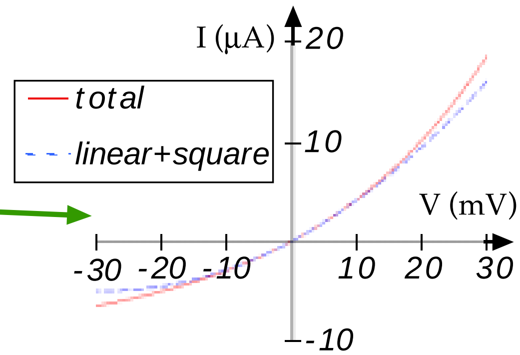
If you want an accurate estimate of average power and need more dynamic range than a bolometer, a thermocouple is your best choice.

How do we measure power? - Diode Sensor

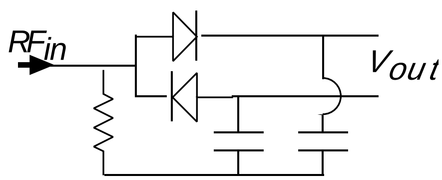


$$i = I_{sat}(e^{\alpha V} - 1)$$

$$i \approx I_{sat}(\alpha V + (\alpha V)^2/2 + \dots)$$



Simplified circuit



Small signal response

Linear cancels

Square law, $V_{DC,out} \propto P_{RF} \propto V_{RF,in}^2$

Large signal, $V_{DC,out} \propto \sqrt{P_{RF}} \propto V_{RF,in}$

Among the items missing from the simplified circuit are resistances in the output section which partly determine the video bandwidth (i.e. how quickly the sensor responds to changes in the envelope power).

Note that the diode doesn't actually respond to specifically to the power or energy. It responds to the voltage, but in a manner (for the square law region) that is proportional to the power.

How do we measure power? - Basic Diode Sensor

Excellent Sensitivity	-70 dBm
Fast Time Response	
Square law range	-70 dBm -> -20 dBm
Large signal range	0 dBm ->

Need to pay attention to linearity issues

Uses
peak power, pulsed signals
avg power - pulsed signal with peak < -20 dBm

If you have a small signal (< -30 dBm) or need to measure modulation properties, such as peak envelope power (PEP), pulse power, or CCDF, you need to use diode based detectors.

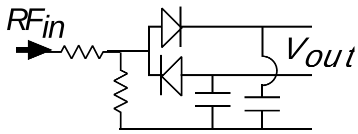
When a thermocouple and diode can be used for the same measurement, the thermocouple is usually the better choice since they have better accuracy.

"Linearity" is most often used to say that $V_{out} \propto P_{abs}$ which means that $V_{out} \propto V_{RF,in}^2$. Thus the diode is considered to be responding linearly in the "square law region", but not the large signal region.

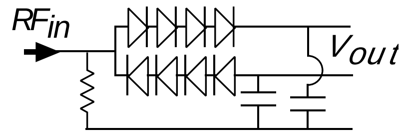
Require power meter for operation to amplify voltage.

How do we measure power? - Diode Sensor Options

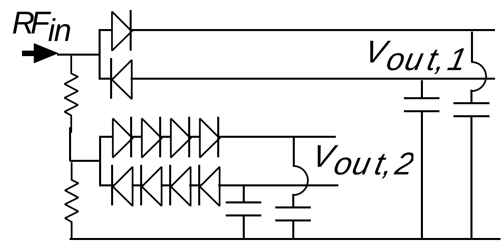
Make better use of the large signal region
1. extended range, CW signal (use power meter to correct for nonlinearity)
2. shift square law response to higher powers



or



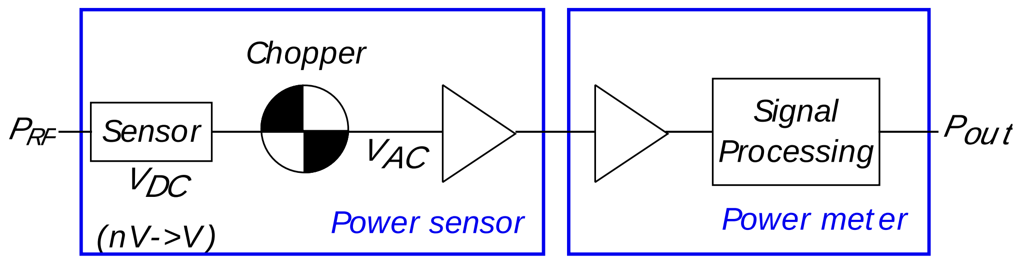
3. look at pulsed signals over wider dynamic range
   fast switch between $V_{out,1}$ and $V_{out,2}$ depending on signal level



A nonlinear response to a CW signal can be corrected after the measurement is made because there is a single function that gives $P_{abs}(V_{out})$. However, this is not true if there is more than one frequency present or if there is any modulation. In these cases, you need to operate in the square law ("linear") regime.

The square law response can be shifted to higher powers using a voltage divider on the input or by replacing a single diode with a stack of diodes in series.

How do we measure power?
Thermoelectric and Diode Power Meters



$P_{out}/P_{RF,inc}$ depends on

1) sensor
2) power meter settings

Want $P_{out}/P_{RF,inc} = 1$

Signal processing includes CF, zeroing, nonlinear and temperature corrections, modulation analysis

(Not simply comparing a DC power and RF power as with bolometers)

This diagram is appropriate for newer style power meters and sensors. With these units, sensors typically store information in EEPROM that is read by the meter.
In older models, the equivalent of a “signal processing” box would be setting the calibration factor (by hand).

I’m only considering power meters for thermocouple and diode detectors. Power meters for thermistor detectors won’t be discussed in any detail.

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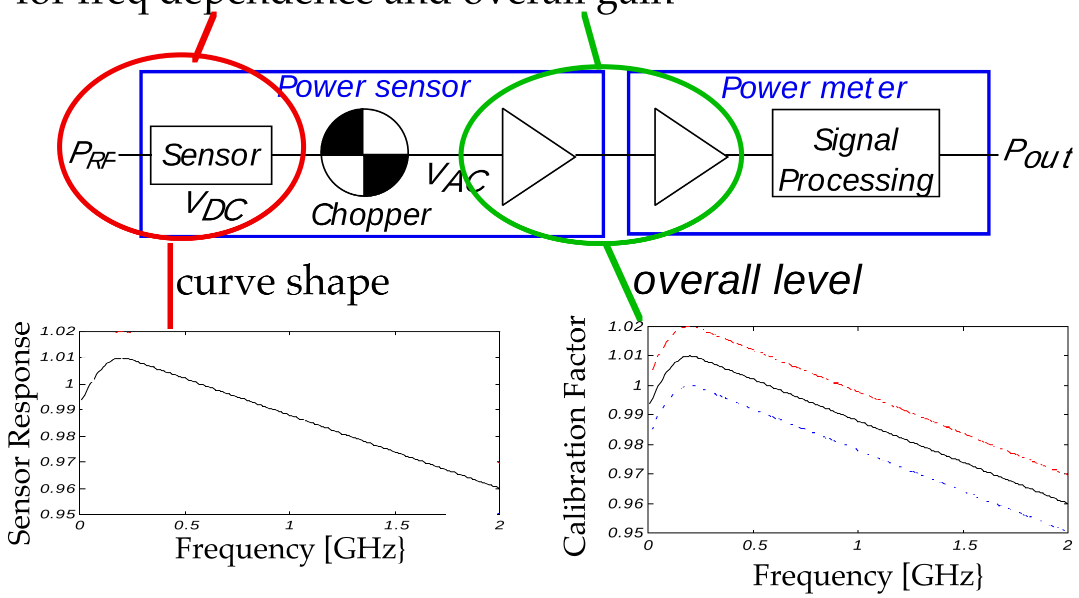
Calibration Factor - Thermoelectric and Diodes

Calibration Factor (CF) used by power meter to compensate for freq dependence and overall gain



There is a frequency dependence in the front-end sensor for both diodes and thermoelectrics that can generally be assumed to be stable for periods of order 1 year. Devices in use for longer periods of time may need to be re-calibrated. The chopping and amplification processes are not frequency dependent.

The AC amplification affects the response of the system equally at all frequencies and is equivalent to moving the response curve up and down as in the right figure.

In order for the meter to have $P_{out} = P_{RF,inc}$, corrections must be applied for the sensor frequency dependence and for the AC amplification. This is called the calibration factor because it plays the same role as the calibration factor in thermistor measurements. One difference is that for a thermistor Type IV power meter, multiplication by a calibration factor occurs as part of the user's post-processing, while for thermoelectric and diode power meters, the calculation is performed internally by the power meter.

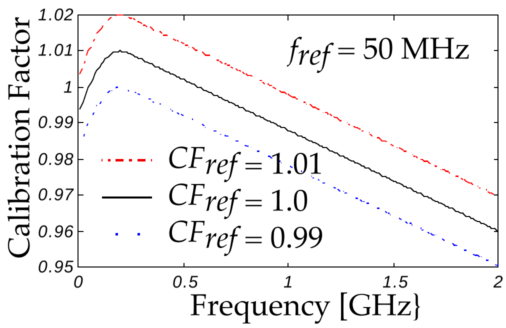
The term "effective efficiency" (η_e) is sometimes used with thermoelectrics and diodes although it is not an efficiency in the same sense as a substituted power (thermistor) measurement. If it is used, then $CF = \eta_e(1-|\Gamma|^2)$ and the response of the system to $P_{RF,net}$ is desired.

Calibration Factor - Thermoelectric and Diodes

Use reference frequency to pin down overall level.
Calibrate with known power at f_{ref}.

Other frequencies: response relative to f_{ref} (using $CF(f)$).

Important quantity is $CF(f)\ /\ CF_{ref}$



New meters - $CF(f_{ref}) = CF_{ref} = 1$

Old meters - $CF_{ref} <= 1$
(some require $CF(f) < 1$ for all f)

Must use consistent CF_{ref} and $CF(f)$!

NIST calibration always performed with $CF_{ref} = 1$.

The calibration factor curve in an older power meter can be adjusted up and down to any desired value. Your reference calibration needs to be done using the same curve as measurements at other frequencies.

NIST always uses a reference calibration factor = 1 in any measurement we report to you.

Measurement Examples - Calibration Factor

NIST calibration data

f	50 MHz	200 MHz	1 GHz	2 GHz
CF	1	1.01	0.988	0.96

Simplest option:

Use a reference cal factor = 1 during setup and NIST values at all frequencies

What if you have an old power meter that needs $CF(f) < 1$

Other Option:

Use reference cal factor = 0.99 in setup and $CF = 0.99\ CF_{NIST}$ at all frequencies

f	50 MHz	200 MHz	1 GHz	2 GHz
CF	0.99	0.9999	0.97812	0.9504

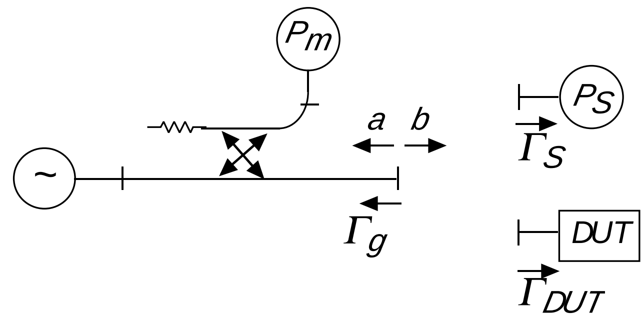
This example is mostly an issue for users with older power meters (or perhaps software) that does not allow $CF > 1$. Newer power meters automatically set the reference cal factor for you and it is generally 1.

Calibration Factor - Thermoelectric and Diodes

Old meters - set calibration factor by hand

New meters - EEPROM used to store calibration factors, temperature sensitivity, nonlinear corrections
Still have to enter frequency

Mismatch Factor

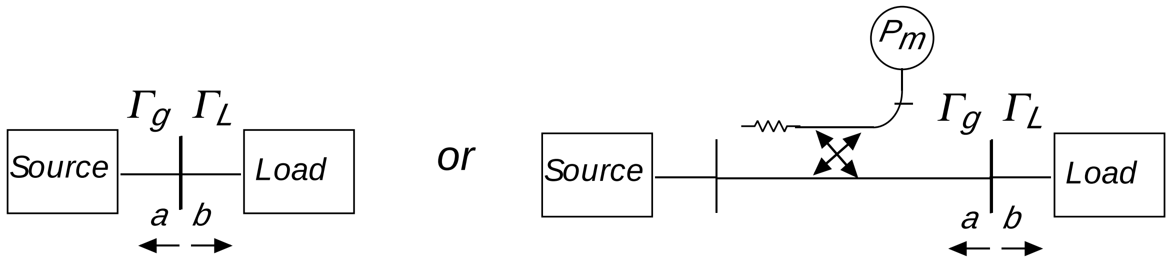


Mismatch factor has corrections for two effects

1. For same P_m, $P_{inc} = |b|^2$ different for standard (P_S) and DUT. (Source loading effect)
2. Difference between incident and net power. $(1-|\Gamma|^2)$

Mismatch factor is often the largest uncertainty in a power measurement.

Mismatch Factor



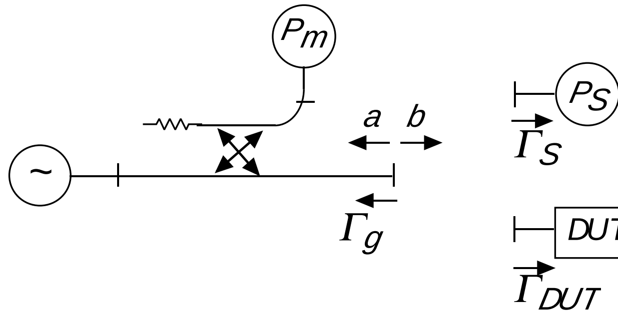
1. Model source loading effects using Γ_g - equivalent generator reflection coeff
   $b = b_g + a\Gamma_g$ b_g is amplitude to matched load (source property)
   $P_{inc} = |b|^2 = |b_g|^2 \,/\, |1 - \Gamma_g \Gamma_L|^2$ also used $a = b\Gamma_L$
   Standing Wave pattern
   Constructive (destructive) interference increases (decreases) power

2. Net power to load = incident power X $(1 - |\Gamma_L|^2)$

$$P_{net} = |b|^2 - |a|^2 = |b_g|^2 \frac{1 - |\Gamma_L|^2}{|1 - \Gamma_g \Gamma_L|^2}$$

The net power relation and various mismatch formulas are derived in the appendix.

Measurement Examples - *Mismatch Factor*



1. Connect standard, set generator so that P_m reads 100 µW
2. Read $P_S = 9.5$ mW
3. Disconnect standard, Connect DUT,
   set generator so that P_m reads 100 µW

How much net power is delivered to DUT?

The power delivered will not be 9.5 mW.

1. The power standard is assumed to measures incident power, not net power. This is most often the case.
2. Mismatch effects cause the incident power in the two cases to be different.

Measurement Examples - *Mismatch Factor*

incident power to standard

$$P_S = \frac{|b_g|^2}{|1-\Gamma_g\Gamma_S|^2}$$

net power to DUT

$$P_{net,DUT} = |b_g|^2 \frac{1-|\Gamma_{DUT}|^2}{|1-\Gamma_g\Gamma_{DUT}|^2}$$

substitute for b_g in $P_{net,DUT}$ using P_S equation

$$P_{net,DUT} = P_S\ MM \quad \text{where mismatch factor} \quad MM = \frac{|1-\Gamma_g\Gamma_S|^2}{|1-\Gamma_g\Gamma_{DUT}|^2}\left(1-|\Gamma_{DUT}|^2\right)$$

plug in numbers -

Let $\Gamma_S = 0.1\exp(j\pi/4)$, $\Gamma_g = 0.25\exp(-j\pi/2)$, $\Gamma_{DUT} = 0.2\exp(j\pi)$

$$MM = \frac{0.9653}{1.0025}\,0.96 = 0.9243$$

$$P_{net,DUT} = 9.5 \times 0.9243 = 8.78\ mW$$

If you are using a thermistor sensor, the calibration factor is $\eta_e(1-|\Gamma_S|^2)$ where η_e is the effective efficiency.

Note that you don't need to calculate $|b_g|^2$.

$P_{net,DUT}$ differs from P_S by 7.6%

Mismatch Factor

Typically, mismatch factor is ratio between net powers
(different from my example)

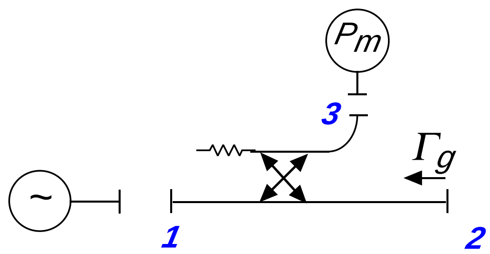
$$MM = \frac{P_{net,A}}{P_{net,B}} = \frac{\left|1-\Gamma_g\Gamma_B\right|^2}{\left|1-\Gamma_g\Gamma_A\right|^2}\,\frac{1-\left|\Gamma_A\right|^2}{1-\left|\Gamma_B\right|^2}$$

Also will see

$$MM = \frac{P_{inc,A}}{P_{inc,B}} = \frac{\left|1-\Gamma_g\Gamma_B\right|^2}{\left|1-\Gamma_g\Gamma_A\right|^2}$$

You need to know how you are transferring the power measurement in order to know which mismatch factor to use.

How do you measure Γ_g?



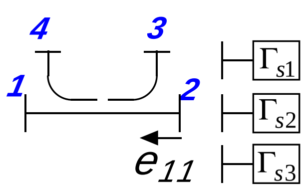
Characterize splitter/coupler (not source)

$$\Gamma_g = S_{22} - \frac{S_{21}S_{32}}{S_{31}}$$

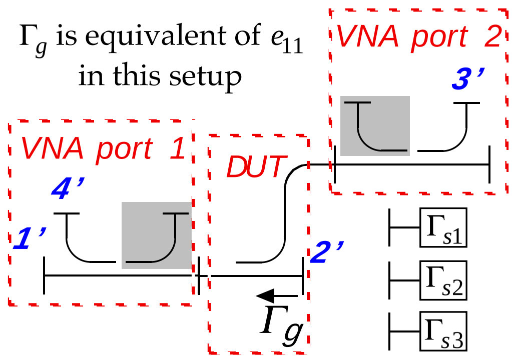
Does not depend on source

(ratio of P_2 to P_3 is important)

VNA one port calibration yields source match term, e_{11} (and e_{00}, Δ_e)



Γ_g is equivalent of e_{11} in this setup



The best approaches for measuring Γ_g involve the use of an intermediate component so that you in fact do not characterize a signal generator.

Reference 6 has details on how the measurement in the lower right is to be done.

There are other techniques for measuring Γ_g as well.

For power measurements, it is generally advantageous to remove the source from the problem. That isn't true for all microwave applications (e.g. noise measurements).

Measurement Techniques - Uncertainty Example

Same example as before, assume uncertainties are independent:

$$P_D \equiv P_{net,DUT} = P_S\ MM;$$

$$u_{P_D} = \sqrt{\left(\frac{\partial P_D}{\partial P_S} u_{P_S}\right)^2 + \left(\frac{\partial P_D}{\partial MM} u_{MM}\right)^2 + u'^2_{other}}$$

or

$$\frac{u_{P_D}}{P_D} = \sqrt{\left(\frac{u_{P_S}}{P_S}\right)^2 + \left(\frac{u_{MM}}{MM}\right)^2 + u^2_{other}}$$

Simplify notation by substituting P_D for $P_{net,DUT}$

Notation: u_x is the uncertainty in x

u_{P_D} depends on the uncertainties of the underlying measurements, u_{P_S} and u_{MM} as well as the sensitivity of P_D to those measurements through $\partial P_D/\partial P_S$ and $\partial P_D/\partial MM$.

u_{other} includes instrumentation errors in the monitor measurement since these will cause the real value of P_m to differ during the standard and DUT measurements. It also includes other errors that aren't in u_{P_S} or u_{MM}.

Note that $u_{other} = P_D\ u'_{other}$

Each of the terms in the rms sum is itself a sum of terms

Measurement Techniques - Uncertainty Example

Mismatch Uncertainty:
$$MM = \frac{|1-\Gamma_g\Gamma_S|^2}{|1-\Gamma_g\Gamma_{DUT}|^2}\left(1-|\Gamma_{DUT}|^2\right)$$

u_{MM} has 6 separate terms:, $u_{|\Gamma_S|}$, u_{θ_S}, $u_{|\Gamma_{DUT}|}$, $u_{\theta_{DUT}}$, $u_{|\Gamma_g|}$, u_{θ_g}

$$u_{MM} = \sqrt{\left(\frac{\partial MM}{\partial \theta_S}u_{\theta_S}\right)^2 + \left(\frac{\partial MM}{\partial |\Gamma_S|}u_{|\Gamma_S|}\right)^2 + 4\ terms}$$

$$\frac{\partial MM}{\partial \theta_S} = MM\ \frac{2\,|\Gamma_g|\,|\Gamma_S|\sin(\theta_g+\theta_S)}{|1-\Gamma_g\Gamma_S|^2}$$

plug in numbers - all $u_\theta = 2°$, all $u_{|\Gamma|} = 0.01$, and same Γ_S, Γ_g, and Γ_{DUT} as before

$$\frac{u_{MM}}{MM} = 0.0084$$

The mismatch depends on 3 complex numbers and therefore 6 parameters.

Reference 7 has a detailed mismatch uncertainty calculation.
In the appendix, $\frac{\partial MM}{\partial|\Gamma_S|}$ and $\frac{\partial MM}{\partial \theta_S}$ are derived. The key to getting these equations is to write MM in terms of the magnitudes and phases.
$u_{|\Gamma_S|}$ and u_{θ_S} are obtained from VNA uncertainty analysis.

Recall that $\Gamma_S = 0.1\exp(j\pi/4)$, $\Gamma_g = 0.25\exp(-j\pi/2)$, $\Gamma_{DUT} = 0.2\exp(j\pi)$

Measurement Techniques - Uncertainty Example

Uncertainty in P_S:

$$\frac{u_{P_S}}{P_S} = \sqrt{\left(\frac{u_{CF}}{CF}\right)^2 + u_{PM}^2 + u_r^2 + ?}$$

Cal Factor (Type B) Power meter (Type B) Repeatability (Type A)

plug in numbers - $u_{CF} = .01$, $CF = 0.98$, $u_{PM} = .007$, $u_r = .001$

$$\frac{u_{P_S}}{P_S} = 0.0124$$

Combined uncertainty - $\frac{u_{P_D}}{P_D} = \sqrt{\left(\frac{u_{P_S}}{P_S}\right)^2 + \left(\frac{u_{MM}}{MM}\right)^2 + u_{other}^2} = 0.0150$

$$P_{net,DUT} = 8.78 \pm 0.13\ mW$$

Uncertainty in the power standard measurement depends on the uncertainty in the calibration factor CF, power meter operation, repeatability, etc. In this example, other contributions (represented by the ?) will be assumed to be negligible.

The combined uncertainty is the combination of mismatch and power standard uncertainties. Other uncertainties are assumed to be negligible.

The uncertainties here are all assumed to be one standard deviation.
In deriving the final values, I have used $u_{MM}/MM = 0.0084$.

Note that in this example, the calibration factor makes the largest contribution to the uncertainty, but that the power meter and mismatch uncertainties are still important. The repeatability uncertainty is relatively small.

Mismatch Factor and Γ_g

What if you don't know phase of Γ_g?

Set $|1-\Gamma_g\Gamma_L|^2 = 1$, with standard deviation of $\sqrt{2}|\Gamma_g||\Gamma_L|$

Same example - $\frac{u_{MM}}{MM} = 0.079$ (over 9x larger) & $\frac{u_{P_D}}{P_D} = 0.080$ (over 5x larger)

Mismatch now dominates

$$P_{net,DUT} = P_S\,(1-|\Gamma_{DUT}|^2) = 9.5 * 0.96 = \mathbf{9.12 \pm 0.73\ mW}$$

(compare to 8.78 ± 0.13 mW)

If you can't measure Γ_g, will get large uncertainty

Low $|\Gamma_g|$ and low $|\Gamma_L|$ will help reduce uncertainty

3 dB pad on source lowers $|\Gamma_g|$ by 6 dB

(assuming pad has low Γ)

low SWR

For a given $|\Gamma_g|$ and $|\Gamma_L|$, the max and min values of $|1-\Gamma_g\Gamma_L|^2$ differ by $4|\Gamma_g||\Gamma_L|$. The $\sqrt{2}$ factor takes into account the U shaped distribution to the complete range of values.

The expression for the uncertainty in this case is in the appendix. The example assumes $|\Gamma_S| = 0.1$, $|\Gamma_g| = 0.25$, and $|\Gamma_{DUT}| = 0.2$. $P_{net,DUT} = 9.12$ mW and $MM = 0.96$ are used for calculations since those are the estimates that would be obtained when the phase isn't known.

If you know Γ_g, there are uncertainty terms proportional to $|\Gamma_g|\Delta|\Gamma_L|$ and $|\Gamma_L|\Delta|\Gamma_g|$. When you don't know the phase of Γ_g, the uncertainty terms are therefore larger by a factor of $|\Gamma|/\Delta|\Gamma|$.

If you don't know either the phase or magnitude of Γ_g, then the uncertainty will be even higher since you will need to make a conservative estimate for $|\Gamma_g|$.

A low SWR is important in a power sensor because it reduces uncertainty due to mismatch.

Measurement Techniques - Uncertainty

Mismatch factor (order 1%)
Calibration Factor (0.2% -> few%)
Reference Power (<~ 1%)

Used to set reference calibration factor for some manufacturers.

Nonlinearity (negligible to few %)

Thermocouple - V response with signal level
Diode - Large signal case is significant problem, even when corrected, there are errors in the correction.

Instrumentation

$\left.\begin{array}{l}\text{Zero Set}\\ \text{Noise}\\ \text{Drift}\end{array}\right\}$ important at low signal level

Human Error

The list is more or less in order of decreasing importance.

If you are using an older power meter without temperature corrections, that can result in larger nonlinearity uncertainties.

Drift uncertainty can be reduced by zeroing the meter close to the time of the measurement.

Measurement Techniques - Human Errors

Using sensor with incorrect bandwidth
with wrong power range

Forgetting to set or setting incorrectly
- frequency - (wrong *CF*)
- *CF* (older meter)
- offset
- reference power
- zero

Outline

1. What is measured (and why?)
   - Why power?
   - CW & Pulsed power parameters
2. How do we measure power?
   - Traceability
   - Sensors: Bolometer
     - Thermocouple
     - Diode
   - Power Meters
3. Measurement Examples and Techniques
   - Calibration Factor
   - Mismatch Factor
   - Uncertainty
4. ***Future***

Future Issues (p. 1)

New user needs / developments

1. Continuing improvement & development of diode detectors to measure new modulation schemes.
2. 40 Gbit/s electro-optic detectors
3. Automotive radar ~ 77 & 95 GHz
4. On-wafer
5. > 100 GHz applications

Time domain vs Frequency Domain

40 Gbit/s electro-optic detectors

Capability being lost

Thermistors - several waveguide models no longer available
can't replace NIST transfer standards

Future Issues (p. 2)

New fundamentals

1. Electro-optic sampling system
   Measure $\boldsymbol{E}_{RF}$ field in coplanar line optically
   Used to calibrate photodiodes
2. Quantum based $\boldsymbol{H}_{RF}$ measurement
   Rabi oscillations in Cs atoms
3. Force - based MEMS detectors

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   This information is also available through "Measurement Method for Determining the Equivalent Reflection Coefficient of Directional Couplers and Power Splitters", Rohde and Schwarz App. Note 1EZ51.
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Biography

Tom Crowley received an S.B. in Physics from MIT and a Ph.D. in Astrophysical Sciences from Princeton University. His graduate work was on microwave scattering measurements of density fluctuations in fusion experiments. For 14 years he was an Assistant and then Associate Professor in the Electrical, Computer, and Systems Engineering Department of Rensselaer Polytechnic Institute. He also was a guest researcher at the University of Texas at Austin and the National Institute for Fusion Science in Nagoya, Japan. During that period, his primary research interests were ion beam diagnostics of fusion plasmas. Since September 2000, he has been a physicist in the Radio Frequency Electronics Group at NIST in Boulder, CO. He is the project leader for power standards and is responsible for the primary power standards in RF and microwaves. His present research interests include calorimeters, bolometric sensors, high power measurements, and quantum-based RF power measurements.

Appendix: Derivation of Mismatch Factor Equations

A.1 Mismatch factor

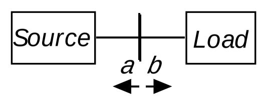
In the example given in lecture, the ratio between the net power to a device under test (DUT) and the power measured by a standard (S) was given as:

$$\frac{P_{net,DUT}}{P_S} = MM2 = \frac{|1-\Gamma_g\Gamma_S|^2}{|1-\Gamma_g\Gamma_{DUT}|^2}(1-|\Gamma_{DUT}|^2) \qquad (1)$$

This formula is valid for the case illustrated in the example. In that case, I was interested in finding the net power delivered to the DUT, but my standard was measuring incident power. If I compare the ratios of the net power delivered to the DUT and net power to the standard, an additional factor of $1-|\Gamma_S|^2$ should be included. The new ratio is given by:

$$\frac{P_{net,DUT}}{P_{net,S}} = MM = \frac{|1-\Gamma_g\Gamma_S|^2}{|1-\Gamma_g\Gamma_{DUT}|^2}\frac{1-|\Gamma_{DUT}|^2}{1-|\Gamma_S|^2} \qquad (2)$$

Equations (1) and (2) are derived below. We begin with an illustration of a generic source-load geometry.



The load will alternately be used to represent a standard and a DUT. We model the loads (L) as passive devices with a reflection coefficient, Γ_L. Therefore,

$$a = \Gamma_L b \qquad (3)$$

The incident power on the load is $|b|^2$ and the net power delivered to the load is

$$P_{net} = |b|^2 - |a|^2 = |b|^2(1-|\Gamma_L|^2). \qquad (4)$$

We model the source output as

$$b = b_g + a\Gamma_g. \qquad (5)$$

The first term, b_g, is the output of the source into a matched load, i.e. $\Gamma_L = 0$. The second term is the change in source output due to loading. a represents a reflected wave traveling from the load to the source. The change in source output is modeled as a 2nd reflection of that wave. By substituting from equation (3) into equation (5), we get the following relationship between b and b_g:

$$b = \frac{b_g}{1-\Gamma_g\Gamma_L}. \qquad (6)$$

Note that depending of the relative phases of Γ_g and Γ_L, the source output can be greater or smaller than its value into a matched load. If we now substitute this expression for b into equation (4), we get the net power delivered to the load,

$$P_{net} = |b_g|^2\frac{1-|\Gamma_L|^2}{|1-\Gamma_g\Gamma_L|^2}. \qquad (7)$$

The mismatch factor in equation (2) is used when you need to know the ratio between the net power delivered to the DUT and the net power delivered to the standard. In this case, you obtain:

$$\frac{P_{net,DUT}}{P_{net,S}} = MM = \frac{|1-\Gamma_g\Gamma_S|^2}{|1-\Gamma_g\Gamma_{DUT}|^2}\frac{1-|\Gamma_{DUT}|^2}{1-|\Gamma_S|^2} \tag{8}$$

The typical power sensors used as standards are usually calibrated so that they read incident power, and not net power. In this case, then we want the following ratio:

$$\frac{P_{net,DUT}}{P_{inc,S}} = \frac{P_{net,DUT}}{P_{net,S}/(1-|\Gamma_S|^2)} = MM2 \tag{9}$$

where *MM2* is given by equation (1).

A.2 Uncertainty in mismatch factor - phases known

If the mismatch factor is calculated as in equation (1) or (2) there is an uncertainty associated with uncertainties in the reflection coefficient magnitudes and phases. I'll evaluate the uncertainty in the eq. (1) MM due to $|\Gamma_S|$ and θ_S as an example. A full calculation for eq. (2) is in reference 7.

Begin by rewriting the factor *f* with Γ_S in terms of amplitudes and phases. Then evaluate the amplitude using its real and imaginary parts:

$$f = \left|1-\Gamma_g\Gamma_S\right|^2 = \left|\,1-|\Gamma_g||\Gamma_S|\cos(\theta_g+\theta_S) - i|\Gamma_g||\Gamma_S|\sin(\theta_g+\theta_S)\,\right|^2 = \left\{1-|\Gamma_g||\Gamma_S|\cos(\theta_g+\theta_S)\right\}^2 + \left\{|\Gamma_g||\Gamma_S|\sin(\theta_g+\theta_S)\right\}^2. \tag{10}$$

This can be further simplified to a form that makes evaluating derivatives easier:

$$f = 1+|\Gamma_g|^2|\Gamma_S|^2 - 2|\Gamma_g||\Gamma_S|\cos(\theta_g+\theta_S). \tag{11}$$

The derivatives of *f* with respect to Γ_S and θ_S are:

$$\frac{\partial f}{\partial|\Gamma_S|} = 2|\Gamma_g|\left\{|\Gamma_g||\Gamma_S| - \cos(\theta_g+\theta_S)\right\} \text{ and } \frac{\partial f}{\partial\theta_S} = 2|\Gamma_g||\Gamma_S|\sin(\theta_g+\theta_S)\ . \tag{12}$$

The results in equation (12) can then be used to find the derivatives of MM with respect to Γ_S and θ_S.

$$\frac{\partial MM}{\partial|\Gamma_S|} = \frac{\partial MM}{\partial f}\frac{\partial f}{\partial|\Gamma_S|} = MM\,\frac{2|\Gamma_g|\left\{|\Gamma_g||\Gamma_S| - \cos(\theta_g+\theta_S)\right\}}{\left|1-\Gamma_g\Gamma_S\right|^2} \tag{13}$$

and

$$\frac{\partial MM}{\partial\theta_S} = MM\,\frac{2|\Gamma_g||\Gamma_S|\sin(\theta_g+\theta_S)}{\left|1-\Gamma_g\Gamma_S\right|^2}. \tag{14}$$

Equation (14) appears in the lecture notes. Similar expressions can be derived for Γ_g and Γ_{DUT}. They are more complicated than eqs. (13) and (14) since they appear in *MM* in more than one place.

A.3 Uncertainty in mismatch factor - phase unknown

If the phase of Γ_L or Γ_g is unknown, then the factor $|1-\Gamma_g\Gamma_L|^2$ can have any value between $(1-|\Gamma_g||\Gamma_L|)^2$ and $(1+|\Gamma_g||\Gamma_L|)^2$. The difference between the minimum and maximum values is $4|\Gamma_g||\Gamma_L|$. The distribution of values is actually peaked at larger values and one standard deviation is given by $\sqrt{2}|\Gamma_g||\Gamma_L|$. The uncertainty in *MM2* in this case is given by:

$$\frac{u_{MM2}}{MM2} = \sqrt{\left(\frac{\sqrt{2}\,|\Gamma_g|\,|\Gamma_S|}{|1-\Gamma_g\Gamma_S|^2}\right)^2 + \left(\frac{\sqrt{2}\,|\Gamma_g|\,|\Gamma_{DUT}|}{|1-\Gamma_g\Gamma_{DUT}|^2}\right)^2 + \left(\frac{-2|\Gamma_{DUT}|\,u_{|\Gamma_{DUT}|}}{1-|\Gamma_{DUT}|^2}\right)^2} \tag{15}$$

where the denominators in the first two factors are approximated as 1. The first two terms will usually be much larger than the last term. The uncertainty in equation (15) is significantly larger than the uncertainty when the mismatch factor is explicitly calculated.