AMPLIFIER AND TRANSISTOR NOISE-PARAMETER MEASUREMENT AT NIST

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Noise Project Areas

- Noise temperature of (1-port) noise sources
- Noise parameters of 2-ports (amplifiers & transistors)
- Remote-sensing radiometer calibration

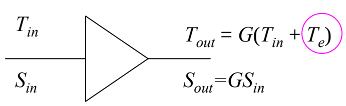
Outline

1. I. Background
2. II. Amplifiers
3. III. Cryogenic Amplifiers
4. IV. On-Wafer Transistors
5. V. Next

I. Background

Noise Parameters

$k_B T \equiv$ available spectral (noise) power



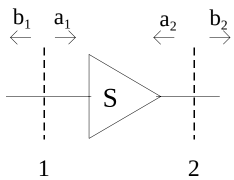
T_e depends on Γ_{in},

$$T_e = T_{e,\min} + t\frac{\left|\Gamma_{opt} - \Gamma_{in}\right|^2}{\left|1+\Gamma_{opt}\right|^2\left(1-\left|\Gamma_{in}\right|^2\right)}$$

4 parameters: $T_{e,min}$, $t = 4R_nT_0/Z_0$, and complex Γ_{opt} (and G)

- To measure:
  - Connect several different terminations with known T_{in}, Γ_{in}
  - Measure T_{out} for each
  - Fit to $T_{out,i} = G[T_{in,i} + T_e(T_{e,\min}, t, \Gamma_{opt})]$

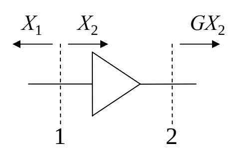
Noise Matrix (Wave Representation)



$$\begin{pmatrix} b_1 \\ b_2 \end{pmatrix} = \begin{pmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} + \begin{pmatrix} c_1 \\ c_2 \end{pmatrix}$$

Intrinsic noise matrix: $N = \langle c_i c_j^* \rangle$ 2×2 hermitian matrix, 4 parameters

$$kX_1 \equiv \langle |c_1|^2 \rangle$$
$$kX_2 \equiv \langle |c_2 / S_{21}|^2 \rangle$$
$$kX_{12} \equiv \langle c_1 (c_2 / S_{21})^* \rangle$$



n.b. $X_2 = T_{e,0}$

- General relationships:

X's → IEEE

$$t = X_1 + |1+S_{11}|^2 X_2 - 2\,\mathrm{Re}\left[(1+S_{11})^* X_{12}\right],$$

$$T_{e,\min} = \frac{X_2 - |\Gamma_{opt}|^2 \left[X_1 + |S_{11}|^2 X_2 - 2\,\mathrm{Re}(S_{11}^* X_{12})\right]}{\left(1+|\Gamma_{opt}|^2\right)},$$

$$\Gamma_{opt} = \frac{\eta}{2}\left(1 - \sqrt{1 - \frac{4}{|\eta|^2}}\right),$$

$$\eta = \frac{X_2\left(1+|S_{11}|^2\right) + X_1 - 2\,\mathrm{Re}(S_{11}^* X_{12})}{(X_2 S_{11} - X_{12})}.$$

IEEE → X's

$$X_1 = T_{e,\min}\left(|S_{11}|^2 - 1\right) + \frac{t|1 - S_{11}\Gamma_{opt}|^2}{|1+\Gamma_{opt}|^2},$$

$$X_2 = T_{e,\min} + \frac{t|\Gamma_{opt}|^2}{|1+\Gamma_{opt}|^2},$$

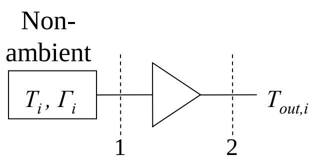
$$X_{12} = S_{11} T_{e,\min} - \frac{t\Gamma_{opt}^*\left(1 - S_{11}\Gamma_{opt}\right)}{|1+\Gamma_{opt}|^2}.$$

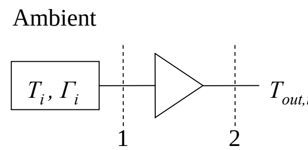
n.b. $X_2 = T_{e,0}$

$X_1 \geq 0$

II. Amplifiers

Measurements

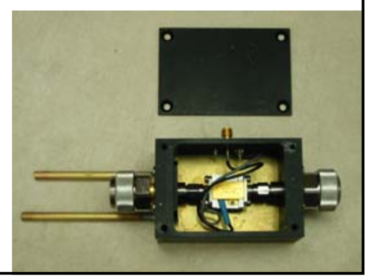




$T_{out} = T_{out}(\Gamma_{\mathrm{i}}, T_{\mathrm{i}}, S_{\mathrm{ij}}, X_1, X_2, X_{12})$

Measure for several (1 hot + 7 ambient)

terminations and fit for X_1, X_2, X_{12} , and G.



Measurement Results

TABLE I

MEASURED VALUES OF THE NOISE PARAMETERS FOR THE AMPLIFIER ALONE.

	8 GHz	9 GHz	10 GHz	11 GHz	12 GHz
X_1(K)	64.5 ± 5.9	67.7 ± 5.5	68.6 ± 5.8	70.4 ± 6.0	80.3 ± 7.0
X_2(K)	110.0 ± 8.4	115.7 ± 7.8	117.6 ± 8.2	124.6 ± 8.4	134.4 ± 10.7
ReX_{12}(K)	9.4 ± 1.5	-7.2 ± 1.2	8.2 ± 1.2	-10.2 ± 1.0	14.3 ± 1.5
ImX_{12}(K)	20.2 ± 1.3	-14.3 ±1.4	10.7 ± 1.5	-14.3 ± 1.6	18.9 ± 2.8
G_0	2031 ± 32	2047 ± 30	1987 ± 30	2121 ± 33	1649 ± 31
T_{min}(K)	112.6 ± 8.4	112.2 ± 7.8	115.1 ± 8.2	123.4 ± 8.4	133.4 ± 10.6
t(K)	128.3 ± 2.6	234.2 ± 3.8	145.8 ± 3.0	223.9 ± 3.9	209.8 ± 4.5
ReΓ_{opt}	-0.172 ± 0.004	0.130 ± 0.004	-0.115 ± 0.005	0.077 ± 0.005	-0.006 ± 0.004
ImΓ_{opt}	0.101 ± 0.008	-0.046 ±. 0.009	-0.003 ± 0.008	-0.004 ± 0.008	-0.069 ± 0.008

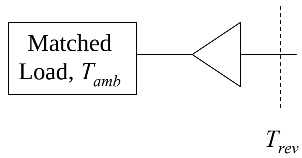
Max(χ^2/DOF) = 0.32

N.b.: in dB,
$G_0 \approx$ 33 dB ± 0.07 dB
$NF \approx$ 1.40 dB ± 0.09 dB

Uncertainties are standard uncertainties (1σ).

Checks & Verification, T_{rev} test

- So what makes us think that we might know what we're doing?
- Have implemented two tests, T_{rev} test and isolator test.
- T_{rev} test: Measure noise temp from input of amplifier, when output is terminated in a matched load.



- Can show that for $\Gamma_L S_{21} S_{12}$ small,

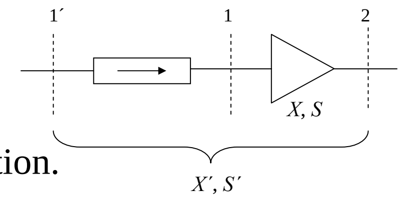
$$T_{rev} \approx \frac{X_1}{\left(1-\left|\Gamma_1\right|^2\right)}$$

- So measure T_{rev}, compare to value predicted from the value of X_1 from the noise-parameter determination. (Use full expression for T_{rev}, not approximation.)
- If working in terms of IEEE parameters, convert to X's to compute T_{rev} & compare to measured value

$$T_{rev} \approx \left[T_{e,\min}\left(\left|S_{11}\right|^2 - 1\right) + \frac{t\left|1 - S_{11}\Gamma_{opt}\right|^2}{\left|1+\Gamma_{opt}\right|^2} \right]\left(1-\left|\Gamma_1\right|^2\right)^{-1}$$

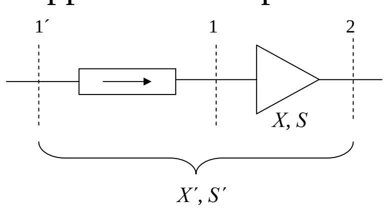
“Isolator” Test

- Connect isolator to amplifier input & measure noise parameters of combination.



- X' parameters can be written in terms of X parameters (amp alone) and the S-parameters of amp and isolator, by using Bosma’s Theorem + standard S-parameter algebra.

- Full expressions are long & ugly; we use them but won’t show them.
- Approximate expressions:



$$X_1' \approx T_I,$$

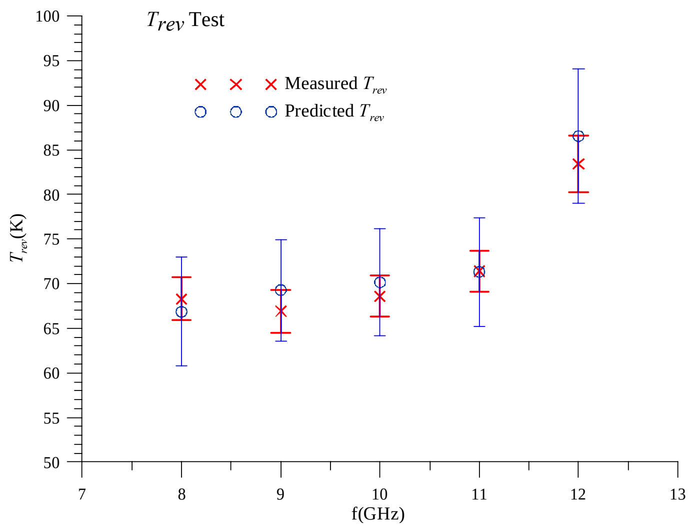
$$X_2' \approx \frac{\left(X_2 + T_I\left(1 - \left|S_{21}^I\right|^2\right)\right)}{\left|S_{21}^I\right|^2},$$

$$X_{12}' \approx \frac{S_{12}^I}{S_{21}^{I*}} X_{12} - T_I S_{11}^I,$$

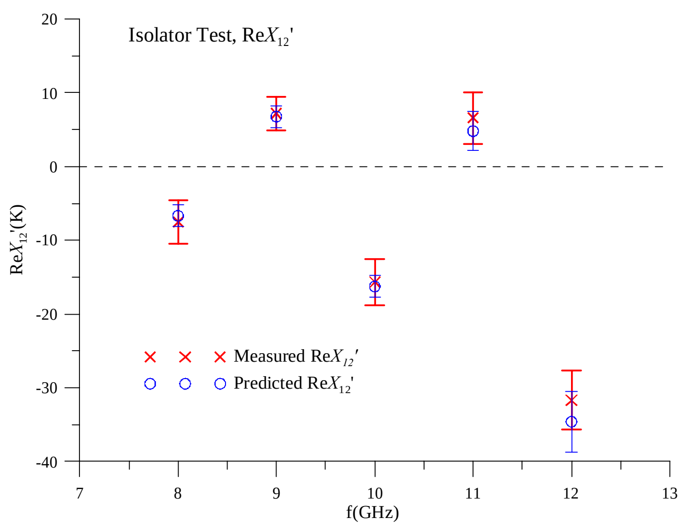
X_{12}' is small and (approximately) independent of amplifier; excellent verification test.

(Could also do with an attenuator in place of the isolator—useful for on wafer.)

Results of Tests (just the interesting ones):



Error bars are standard uncertainties (1σ).



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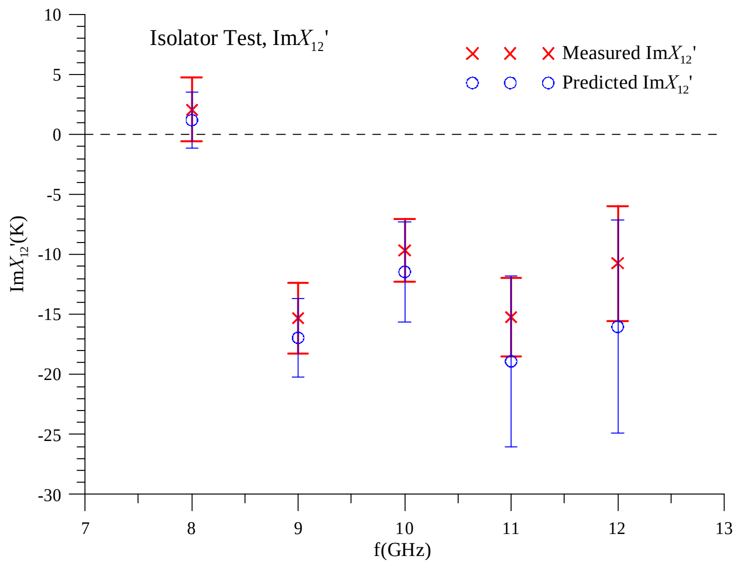


Error bars are standard uncertainties (1σ).

Uncertainties

- Type A (statistical): obtained in the fitting process,

$$u_A(i) = \sqrt{Cov_{ii}}$$

- Type B (other): from Monte Carlo program
- Standard (combined):

$$u_c = \sqrt{u_A^2 + u_B^2}$$

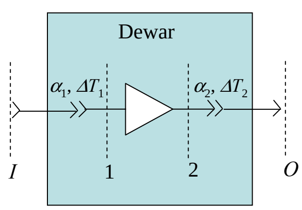
- Also used MC program to
  - Study dependence of noise-parameter uncertainties on underlying uncertainties
  - Simulate & compare different measurement strategies.

III. Cryogenic Amplifier Measurements

(with Eyal Gerecht & Dazhen Gu, THz Tech. Project)

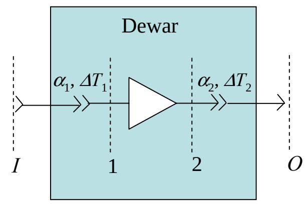
General:

- Want to measure noise temp or noise figure for matched case (T_{50} or F_{50}) at amplifier reference planes (in cryostat).
- Can only access cryostat input & output reference planes.
- Must correct for lines.
- Also potential difficulties due to very low amplifier noise temperature (< 10 K).



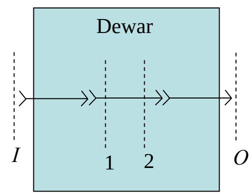
Line Characterizaton:

- Must determine two parameters for each line.

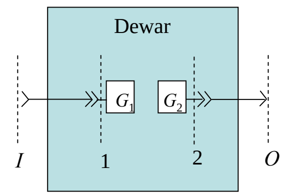


$$T_O = \alpha_2 T_2 + \Delta T_2$$
$$T_1 = \alpha_1 T_I + \Delta T_1$$

- Use 6 measurements:



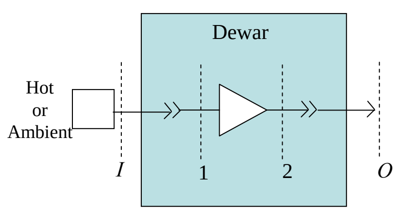
Hot & ambient on *I*, measure *T* at *O*.
Hot & ambient on *O*, measure *T* at *I*.

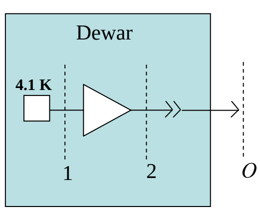


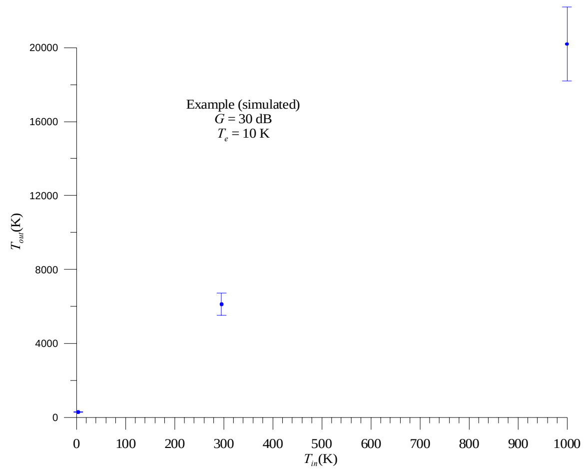
4.1 K at 1 & 2, measure *T* at *I* & at *O*.

3-Point Y-Factor Method (hot-ambient-cold):

- Two measurements through input port (hot & ambient sources) + one measurement with matched load connected directly to amplifier, inside cryostat (cold source, 4.1 K)





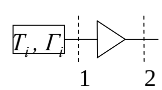


Preliminary results:

	T_{50}(K)	G(dB)
1 GHz	7.3 ± 1	33.1
2 GHz	5.5 ± 1	34.0

IV. On-Wafer Noise Parameters

- Working with IBM & RFMD
- Complications on wafer:
  - characterizing & correcting for probes
  - loss through probes → can't get too close to $|\Gamma| = 1$ for input terminations
- Device properties problematic:
  - poorly matched: $|S_{11}|$, $|S_{22}|$, $|\Gamma_{opt}| > 0.5$ ⇒possible power transfer problems; non-physical results more common
  - $|S_{12}S_{21}|$ not very small → $|\Gamma_2| > 1$ in some cases
  - very low noise, $T_e \sim 10 – 30$ K
  - not necessarily stable



Bounds & Unphysical Results

- Two most commonly violated bounds are

$$X_1 = T_{e,\min}\left(|S_{11}|^2 - 1\right) + \frac{t\left|1 - S_{11}\Gamma_{opt}\right|^2}{\left|1 + \Gamma_{opt}\right|^2} \geq 0$$

$$T_{e,\min} = \frac{X_2\left(1 - \left|S_{11}\Gamma_{opt}\right|^2\right) - \left|\Gamma_{opt}\right|^2\left[X_1 - 2\,\mathrm{Re}\left(S_{11}^* X_{12}\right)\right]}{\left(1 + \left|\Gamma_{opt}\right|^2\right)} \geq 0$$

- Have done simulations to see how often one obtains unphysical results
- Results depend on everything, of course, but representative results are:

Input Uncertainties (correlated & uncorrelated)

	Good	Less Good
Γ(<0.5), cor	.0021	.003
Γ(<0.5), uncor	.0021	.0021
Γ(>0.5), cor	.0035	.004
Γ(>0.5), uncor	.0035	.0035
S_{21}, cor	.007	.007
S_{21}, uncor	.007	.007
Connector repeatability	.001	.001
$T_{ambient}$, cor	0.5 K	0.5 K
$T_{ambient}$, uncor	0.0	0.5 K
T(not near ambient), cor	0.7 %	1.0 %
T(not near ambient), uncor	0.7 %	0.7 %

Percent unphysical results

	Good Uncerts	Less Good
Device 1	0.9 %	1.4 %
Device 2	9.4 %	11.5%

In Progress with IBM & RFMD

- Measure S-parameters & noise parameters of MOSFETs with 0.13 µm gate length; inter-compare.
- Evaluate our uncertainties, and theirs (?).
- Incorporate measurement of T_{rev} on wafer.
- Apply checks on wafer:
  - compare (attenuator + transistor) to (transistor)
  - noise parameters of mismatched attenuator
  - T_{rev}

V. Next

- Amplifier noise-parameter comparison with NPL
- Further measurements on cryogenic amplifier with THz Project
- Continue on-wafer work with IBM & RFMD
- Considerable amount of data to analyze, document

- Automate measurements & data handling
- Improve measurements—cold input source, better choice of input terminations, …
- Round-robin with industry?

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