

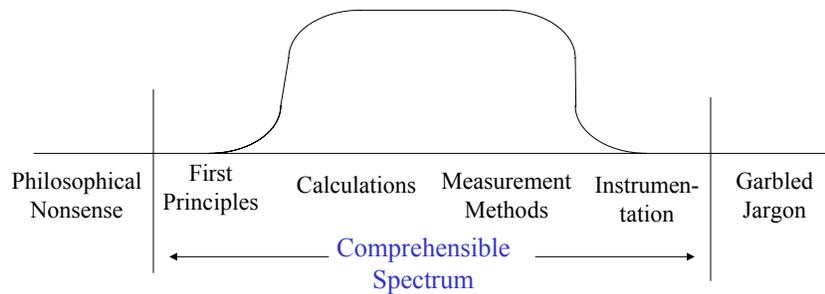
# THERMAL NOISE MEASUREMENTS

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## INTRODUCTION

- General Content:



- Outline
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    - Noise Temperature Definition
    - Microwave Networks & Noise
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    - Total-power radiometer
      - general
      - simple, idealized case
      - not so simple case
    - Uncertainties
      - simple case
      - not so simple case—not today
    - Adapters

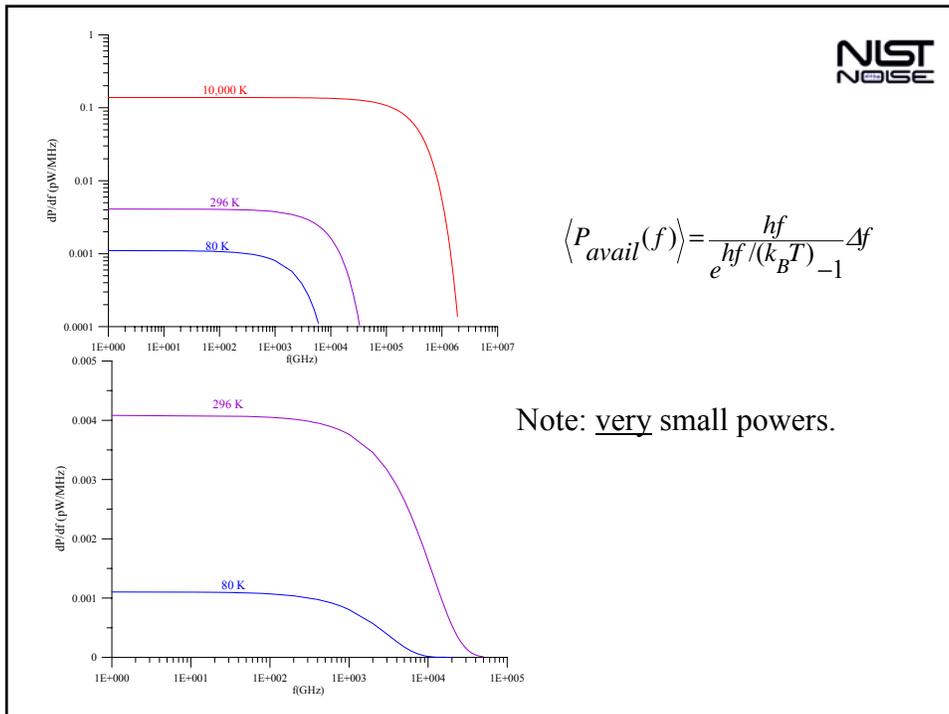
- Outline (cont'd)
  - Noise Figure & Parameters
    - Noise Figure defined
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# I. BASICS

## Nyquist Theorem

- Derivation:
  - Electr. Eng. [1-4]
  - Physics, Stat. Mech. [4]
- For passive device, at physical temperature T, with small  $\Delta f$ ,

$$\langle P_{avail}(f) \rangle = \frac{hf}{e^{hf/(k_B T)} - 1} \Delta f$$



- Limits
  - small  $f$ :  $\langle P_{avail} \rangle \approx k_B T \Delta f [1 - hf/(2k_B T)]$   
 $\approx k_B T \Delta f$
  - large  $f$ :  $\rightarrow 0$
  - knee occurs around  $f(\text{GHz}) \approx 20 T(\text{K})$
- Quantum effect
  - $h/k_B = 0.04799 \text{ K/GHz}$
  - So at 290 K, 1 % effect at 116 GHz  
 at 100 K, 1 % effect at 40 GHz  
 at 100 K, 0.1 % effect at 4 GHz  
 30 K @ 40 GHz  $\rightarrow$  6.4%, 0.26 dB

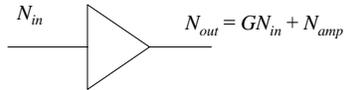
## NOISE TEMPERATURE

- What about active devices? Can we define a noise temperature?
- Several different definitions used:
  - delivered vs. available power
  - with or without quantum effect  
*i.e.*, does  $T_{noise} \propto P_{avail}$  (“power” definition), or  
 is  $T_{noise}$  the physical temperature that would result in that value of  $P_{avail}$  (“equivalent-physical-temperature” definition)?

- IEEE [5]: “(1)(general)(at a pair of terminals and at a specific frequency) the temperature of a passive system having an available noise power per unit bandwidth equal to that of the actual terminals.”  
and  
“(4)(at a port and at a selected frequency) A temperature given by the exchangeable noise-power density divided by Boltzmann’s constant, at a given port and at a stated frequency.”

- We (I) will use second definition, noise temp  $\equiv$  available spectral noise-power divided by Boltzmann’s constant.
- It is the common choice in international comparisons [6] and elsewhere [7].
- It is much more convenient for amplifier noise considerations (at least for careful ones)

Note: In fact there is an additional noise contribution due to vacuum fluctuations, e.g., at the input of an amplifier,  $T_{vac} = hf/(2k_B)$ , but it is *almost* always negligible at microwave frequencies ( $T_{vac} = 0.24$  K at 10 GHz). Ref. [7] discusses one way of treating  $T_{vac}$ .



If  $N = kT$ , then  $N_{in} = kT_{in}$ , etc., and  $T_{out} = GT_{in} + T_e$

But if we use the “equivalent physical temperature” definition, then  $N_{in} = \frac{hf}{e^{hf/kT_{in}} - 1}$   
and similarly for the others, and so  $\frac{hf}{e^{hf/kT_{out}} - 1} = G \left( \frac{hf}{e^{hf/kT_{in}} - 1} + \frac{hf}{e^{hf/kT_e} - 1} \right)$ .

Solving for  $T_{out}$ , we would get

$$T_{out} = \frac{hf}{k} \left\{ \ln \left[ 1 + \frac{1}{G} \left( \frac{1}{e^{hf/kT_{in}} - 1} + \frac{1}{e^{hf/kT_e} - 1} \right) \right] \right\}^{-1}$$

- So  $P_{avail} = k_B T_{noise} \Delta f$
- And for passive devices,

$$T_{noise} = \frac{1}{k_B} \left[ \frac{hf}{e^{hf/(k_B T)} - 1} \right] \approx T_{phys}$$

- Convenient to define “Excess noise ratio”

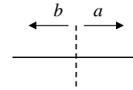
$$ENR_{(avail)} \equiv \frac{T_{(avail)} - T_0}{T_0} \quad T_0 = 290K$$

$$T=9500 K \Rightarrow ENR \approx 15.02 \text{ dB}$$

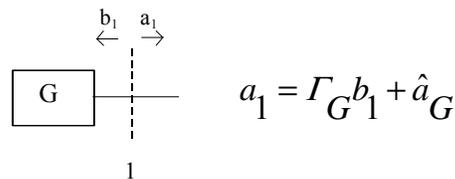
No matter what definition of noise temperature you choose,  
it is helpful to **state your choice**.

## MICROWAVE NETWORKS & NOISE [8,9]

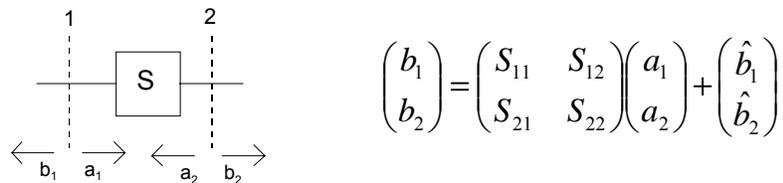
- Assume lossless lines, single mode.
- Travelling-wave amplitudes  $a, b$ .
- Normalized such that  $P_{del} = |a|^2 - |b|^2$  is spectral power density.
- May be a little careless about B; assume that it's 1Hz where needed.



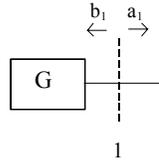
- Describe (linear) one-ports by



- And (linear) two-ports by



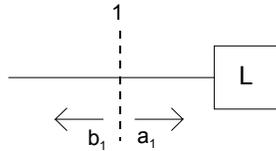
- Available power:



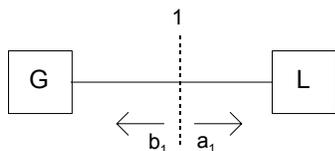
$$P_G^{avail} = \frac{|\hat{a}_G|^2}{1 - |\Gamma_G|^2}$$

$$\langle |\hat{a}_G|^2 \rangle = (1 - |\Gamma_G|^2) k_B T_G$$

- Delivered power:



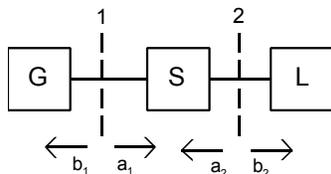
$$P_1^{del} = |a_1|^2 - |b_1|^2 = |a_1|^2 (1 - |\Gamma_L|^2)$$



Mismatch Factor

$$M_1 = \frac{P^{del}}{P^{avail}} = \frac{(1 - |\Gamma_L|^2)(1 - |\Gamma_G|^2)}{|1 - \Gamma_L \Gamma_G|^2}$$

Efficiency

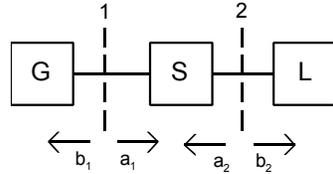


$$\eta_{21} = \frac{P_2^{del}}{P_1^{del}} = \frac{|S_{21}|^2 (1 - |\Gamma_L|^2)}{|1 - \Gamma_L S_{22}|^2 (1 - |\Gamma_{S_L}|^2)}$$

$$= \frac{|S_{21}|^2 (1 - |\Gamma_L|^2)}{|1 - \Gamma_L S_{22}|^2 - |(S_{12} S_{21} - S_{11} S_{22}) \Gamma_L + S_{11}|^2}$$

- Available power ratio:

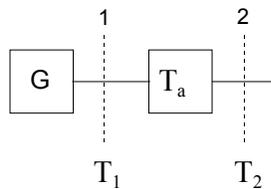
$$\alpha_{21} \equiv p_{2,avail}/p_{1,avail} (\hat{b}_1, \hat{b}_2 = 0)$$



$$\alpha_{21} = \frac{|S_{21}|^2 (1 - |\Gamma_G|^2)}{|1 - \Gamma_G S_{11}|^2 (1 - |\Gamma_{GS}|^2)}$$

$$\Gamma_{GS} = S_{22} + \frac{S_{12} S_{21} \Gamma_G}{1 - \Gamma_G S_{11}}$$

- Temperature translation through a passive, linear, 2-port (attenuator, adapter, line, ...)



$$P_2^{avail} = \alpha_{21} P_1^{avail} + f_0(T_a)$$

$$T_2 = \alpha_{21} T_1 + f(T_a)$$

Say  $T_1 = T_a$ , then  $T_2$  must =  $T_a$ , so

$$T_2 = T_a = \alpha_{21} T_a + f(T_a)$$

$$f(T_a) = (1 - \alpha_{21}) T_a$$

and therefore

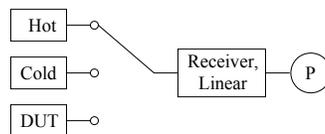
$$T_2 = \alpha_{21} T_1 + (1 - \alpha_{21}) T_a$$

## II. NOISE-TEMPERATURE MEASUREMENT

### Total-Power Radiometer [10-12]

- Radiometer: measures “radiated” power. For us, measures delivered power (in w.g. or transmission line), & we convert to delivered power & therefore to noise temperature.
- Two principal types of radiometer for noise-temperature measurements are Dicke radiometer and total-power radiometer [10].
- Total-power radiometer is most common for lab use, & that’s what we’ll discuss.

- Simple case: symmetric, matched (all  $\Gamma$ 's = 0)



Matched  $\rightarrow p_{del} = p_{avail}$       Linear  $\rightarrow P = a + bp_{del} = a + bp_{avail}$

2 standards ( $h, c$ ) determine  $a, b$ :

$$P_h = a + bk_B BT_h$$

$$P_c = a + bk_B BT_c$$

So  $a = P_c - bk_B BT_c$        $Bk_B b = \frac{P_h - P_c}{T_h - T_c}$

Note: This could be written as  
 $T_{out, rec} = T_{rec} + G_{rec} T_{in, rec}$

Then  $T_x = T_c + \frac{(Y_x - 1)}{(Y_h - 1)} (T_h - T_c)$ , where  $Y_x = \frac{P_x}{P_c}$ ,  $Y_h = \frac{P_h}{P_c}$

- Not-so-simple case (unmatched, asymmetric)

Three complications:

- $p_{del} = Mp_{avail}$
- $p_{del}(\text{to rad.}) = \eta p_{del}(\text{from source})$ ,  
and  $\eta_x \neq \eta_h \neq \eta_c$
- $a, b = a(\Gamma), b(\Gamma)$
- Handle first two by measuring and correcting.

- For dependence of  $a$  and  $b$  on  $\Gamma$ , have three choices:
  - tune so that  $\Gamma_h = \Gamma_c = \Gamma_x$  (very narrow frequency range, need special standards)
  - characterize dependence on  $\Gamma$  (broadband, but a lot of work, and difficult to get good accuracy)
  - isolate (easy, accurate, but limits frequency range & difficult at low frequency)

- If isolate,  $a$  and  $b$  are (almost) independent of the source, and

$$T_x = T_{amb} + \left( \frac{M_S \eta_S}{M_x \eta_x} \right) \frac{(Y_x - 1)}{(Y_S - 1)} (T_S - T_{amb})$$

## Uncertainties

- Simple case (matched):

$$T_x = T_a + \frac{(Y_x - 1)}{(Y_S - 1)} (T_S - T_a) \frac{M_h \eta_h}{M_x \eta_x}$$

small uncert, but linearity is a concern  
 about 1 or 2%  
 typically around 1%

Uncert "should" be negligible

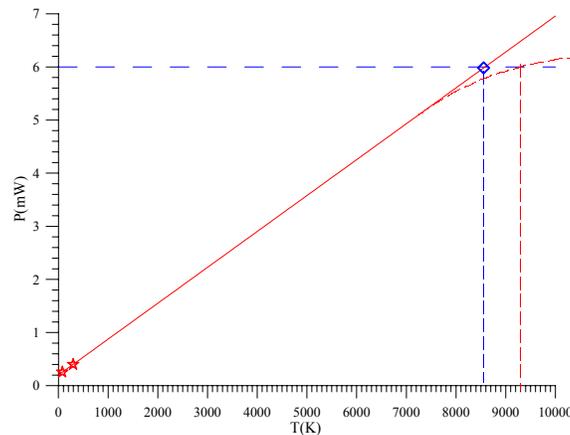
For the ENR, this  $\Rightarrow u(\text{ENR}) \approx 0.10 \text{ dB to } 0.15 \text{ dB}$

- Simple-case uncerts (cont'd)
  - drift: temperature stability/control important (effect minimized by frequent switching to standards)
  - connector variability: hard to do much better than 0.1%, easy to do considerably worse.
  - $\Delta a, \Delta b$  (due to  $\Delta T$ ): depends on details of system, can make a crude estimate:

$$T_{rev} \sim T_e, \quad |\Delta T| \sim 0.05 \text{ or } 0.1$$

$$\text{So } \Delta T_{in} \sim 0.05 \text{ or } 0.1 \times T_e$$

- linearity: serious concern if  $T_x$  very different from standards, less (but some) worry if  $T_x$  near temperature of a standard.



- Uncertainties (more careful case)  
(Numbers are for NIST case) [13,14]

– Radiometer equation:

$$T_x = T_{amb} + \frac{M_S \eta_S (Y_x - 1)}{M_x \eta_x (Y_S - 1)} (T_S - T_{amb})$$

– Ambient standard:

$$\frac{u_{amb}(T_x)}{T_x} = \left| \frac{T_x - T_S}{T_a - T_S} \right| \frac{T_a}{T_x} \varepsilon_{T_a}, \quad \varepsilon_{T_a} = \frac{0.1K}{296K} = 0.034\%$$

– “Other” standard:

$$\frac{u_{T_S}(T_x)}{T_x} = \left| 1 - \frac{T_a}{T_x} \right| \left| \frac{T_S}{T_a - T_S} \right| \frac{u(T_S)}{T_S}, \quad \frac{u(T_S)}{T_S} = 0.2\% (NIST W.G.), 0.8\% (NIST coax)$$

– Path asymmetry: (zero if connect to same port)

$$\frac{u_{\eta/\eta}(T_x)}{T_x} = \left| 1 - \frac{T_a}{T_x} \right| u(\eta/\eta), \quad u(\eta/\eta) = 0.2\% \text{ to } 0.56\%$$

– Mismatch:

$$\frac{u_{M/M}(T_x)}{T_x} = \left| 1 - \frac{T_a}{T_x} \right| u(M/M), \quad u(M/M) \approx 0.2\%$$

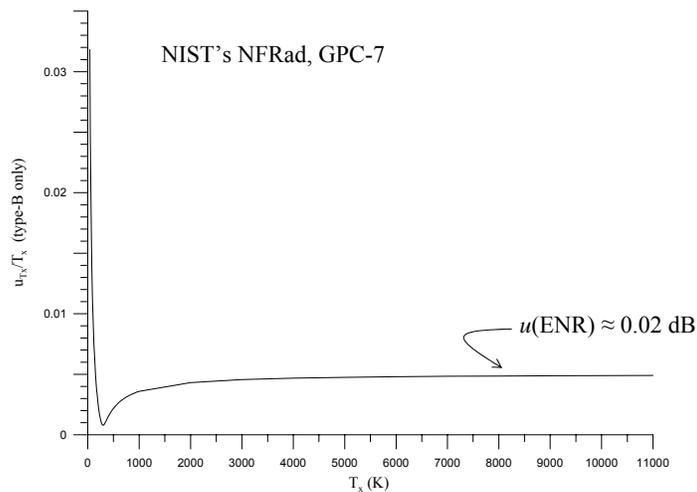
– Connectors:

$$\frac{u_{conn}(T_x)}{T_x} = u_0 \left| 1 - \frac{T_a}{T_x} \right| \sqrt{f(\text{GHz})}, \quad u_0 \approx 0.053\% \text{ to } 0.069\%$$

(depending on connector type)

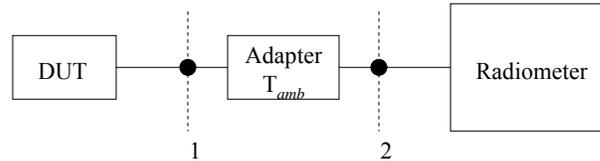
– Other: Nonlinearity, imperfect isolation, power ratio measurement, and broadband mismatch/frequency offset all lead to small (<0.1%) uncertainties for  $T_x$  around 10 000 K (for us/NIST).

- $u_B(T)/T$  as a function of  $T$   
Standard relative uncertainty ( $1\sigma$ )



## Adapters

- Measure T at 2, want T at 1.



$$T_2 = \alpha_{21}T_{DUT} + (1 - \alpha_{21})T_{amb}$$

$$\text{So } T_{DUT} = \frac{T_2 - (1 - \alpha_{21})T_{amb}}{\alpha_{21}}$$

For a good adapter,  $\alpha \approx 0.95 - 0.99$ , depending on frequency.

Determine  $\alpha$  from  $\alpha_{21} = \frac{|S_{21}|^2(1 - |\Gamma_1|^2)}{|1 - \Gamma_1 S_{11}|^2(1 - |\Gamma_2|^2)}$  or [15,16] or ....

## III. NOISE FIGURE & NOISE PARAMETERS

### Noise Figure Defined

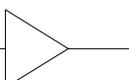
- Want a measure of how much noise an amplifier adds to a signal or how much it degrades the S/N ratio.

- Define Noise Figure, IEEE [17]:  
(at a given frequency) the ratio of total output noise power per unit bandwidth to the portion of the output noise power which is due to the input noise, evaluated for the case where the input noise power is  $k_B T_0$ , where  $T_0 = 290$  K. (vacuum fluctuation comment)

- Noise figure & signal to noise ratio[18]:

$$\frac{(S/N)_{in}}{(S/N)_{out}} = \frac{S_{in}/290K}{GS_{in}/(G \times 290K + N_{amp})} = \frac{G \times 290K + N_{amp}}{G \times 290K} = F$$

- Effective input noise temperature:



$$S_{out} = G S_{in}$$

$$N_{out} = G N_{in} + N_{amp} = G k_B T_{in} + N_{amp}$$

$$\text{Define } N_{amp} \equiv G k_B T_e$$

$$\text{So } N_{out} = G k_B (T_{in} + T_e)$$

So Noise Figure becomes

$$F = \frac{\text{Noise out}}{\text{Noise in}} = \frac{G(T_0 + T_e)}{G T_0} \quad F(\text{dB}) = 10 \log_{10} \left( \frac{T_0 + T_e}{T_0} \right)$$

Note:  $G, F, T_e$  all depend on  $\Gamma_{source}$ .

## Simple Case Measurement, all $\Gamma$ 's equal

$$T_h \rightarrow \left[ G \right] \rightarrow N_{out,h} = Gk_B(T_h + T_e)$$

$$T_c \rightarrow \left[ G \right] \rightarrow N_{out,c} = Gk_B(T_c + T_e)$$

Combine & solve:

$$G = \frac{N_h - N_c}{k_B(T_h - T_c)} \quad T_e = \frac{N_c T_h - N_h T_c}{N_h - N_c} = \frac{T_h - Y T_c}{Y - 1} \quad \text{where } Y = N_h / N_c$$

$$F = 1 + \frac{T_e}{T_0} = 1 + \frac{T_h - Y T_c}{(Y - 1) T_0}$$

In terms of ENR:

$$F = 1 + \frac{T_e}{T_0} = 1 + \frac{T_h - Y T_c}{(Y - 1) T_0} = \frac{ENR}{Y - 1} + \left( \frac{Y}{Y - 1} \right) \left( \frac{T_0 - T_c}{T_0} \right) \approx \frac{ENR}{Y - 1}$$

if  $T_c \approx T_0$  (290 K  $\Rightarrow$   $\sim$  63 °F), then

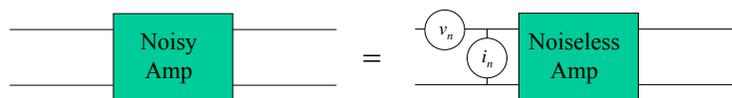
$$F (dB) \approx ENR_h (dB) - (Y - 1)(dB)$$

Advice: Such approximations are useful in conversation or for rough estimates & mental computations. For any “real” computation, use the full, correct expression(s). It only takes a few seconds of extra typing, and it can make a difference in the answer.

## Noise Parameters, IEEE Representation

- But that's just for one value of  $\Gamma_{source}$ . Want to determine  $F$  or  $T_e$  for any  $\Gamma_{source}$ . So parameterize dependence on  $\Gamma_{source}$ .
- Several parameterizations in use; most common are variants of the IEEE [19] form.

- Equivalent circuit:



- (Noise out)/(Noise in) depends on impedance of input termination,  $NF = NF(Z_S)$  or  $NF(\Gamma_S)$ , &  $T_e = T_e(Z_S)$  or  $T_e(\Gamma_S)$ ,

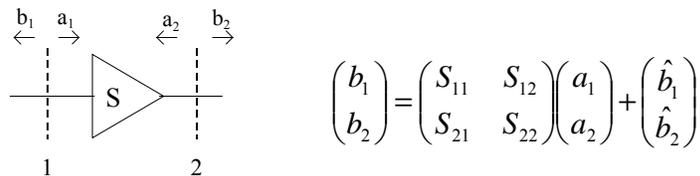
$$NF = NF_{\min} + \frac{4R_n}{Z_0} \frac{|\Gamma_{opt} - \Gamma_S|^2}{|1 + \Gamma_{opt}|^2 (1 - |\Gamma_S|^2)} \quad T_e = T_{e,\min} + t \frac{|\Gamma_{opt} - \Gamma_S|^2}{|1 + \Gamma_{opt}|^2 (1 - |\Gamma_S|^2)}$$

4 parameters:  $T_{e,\min}$ ,  $t = 4R_n T_0 / Z_0$ , and complex  $\Gamma_{opt}$ .

Note: many equivalent forms of IEEE representation; this one is from [20].

## Wave Representation of Noise Matrix

- For microwave radiometry, wave representation [20 – 25] provides more flexibility.
- Linear 2-port:



$$\begin{pmatrix} b_1 \\ b_2 \end{pmatrix} = \begin{pmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} + \begin{pmatrix} \hat{b}_1 \\ \hat{b}_2 \end{pmatrix}$$

- Noise matrix is defined by

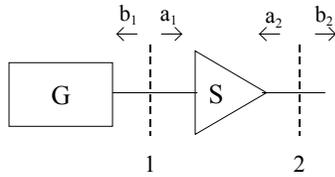
$$N_{ij} = \langle b_i b_j^* \rangle$$

or  $\hat{N}_{ij} = \langle \hat{b}_i \hat{b}_j^* \rangle$  for intrinsic noise matrix

- Four real noise parameters:

$$\langle |\hat{b}_1|^2 \rangle, \langle |\hat{b}_2|^2 \rangle, \langle \hat{b}_1 \hat{b}_2^* \rangle$$

- Output noise temperature  $T_2$



$$k_B T_2 = \frac{|S_{21}|^2}{(1-|\Gamma_{GS}|^2)} [N_G + N_1 + N_2 + N_{12}]$$

$$N_G = \frac{(1-|\Gamma_G|^2)}{|1-\Gamma_G S_{11}|^2} k_B T_G$$

$$N_1 = \left| \frac{\Gamma_G}{1-\Gamma_G S_{11}} \right|^2 \langle \hat{b}_1^2 \rangle$$

$$N_2 = \langle \hat{b}_2 / S_{21} \rangle^2$$

$$N_{12} = 2 \operatorname{Re} \left[ \frac{\Gamma_G}{(1-\Gamma_G S_{11})} \langle \hat{b}_1 (\hat{b}_2 / S_{21})^* \rangle \right]$$

- So for  $T_e$  we have

$$T_e = \frac{|\Gamma_G|^2}{(1-|\Gamma_G|^2)} X_1 + \frac{|1-\Gamma_G S_{11}|^2}{(1-|\Gamma_G|^2)} X_2 + \frac{2}{(1-|\Gamma_G|^2)} \operatorname{Re}[(1-\Gamma_G S_{11})^* \Gamma_G X_{12}]$$

where  $k_B X_1 \equiv \langle \hat{b}_1^2 \rangle$ ,  $k_B X_2 \equiv \langle \hat{b}_2 / S_{21} \rangle^2$ ,  $k_B X_{12} \equiv \langle \hat{b}_1 (\hat{b}_2 / S_{21})^* \rangle$

- Whereas IEEE parameterization is

$$T_e = T_{e,\min} + t \frac{|\Gamma_G - \Gamma_{opt}|^2}{(1-|\Gamma_G|^2) |1 + \Gamma_{opt}|^2}$$

- Can relate the two:

X's → IEEE

$$t = X_1 + |1 + S_{11}|^2 X_2 - 2 \operatorname{Re}[(1 + S_{11})^* X_{12}],$$

$$T_{e,\min} = \frac{X_2 - |\Gamma_{opt}|^2 [X_1 + |S_{11}|^2 X_2 - 2 \operatorname{Re}(S_{11}^* X_{12})]}{(1 + |\Gamma_{opt}|^2)},$$

$$\Gamma_{opt} = \frac{\eta}{2} \left( 1 - \sqrt{1 - \frac{4}{|\eta|^2}} \right),$$

$$\eta = \frac{X_2 (1 + |S_{11}|^2) + X_1 - 2 \operatorname{Re}(S_{11}^* X_{12})}{(X_2 S_{11} - X_{12})}.$$

IEEE → X's

$$X_1 = T_{e,\min} (|S_{11}|^2 - 1) + \frac{t |1 - S_{11} \Gamma_{opt}|^2}{|1 + \Gamma_{opt}|^2},$$

$$X_2 = T_{e,\min} + \frac{t |\Gamma_{opt}|^2}{|1 + \Gamma_{opt}|^2},$$

$$X_{12} = S_{11} T_{e,\min} - \frac{t \Gamma_{opt}^* (1 - S_{11} \Gamma_{opt})}{|1 + \Gamma_{opt}|^2}.$$

Notes:

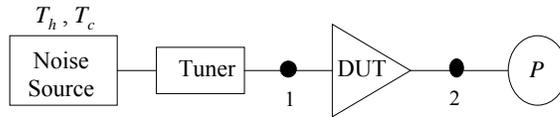
$$X_2 = T_{e,0}$$

Bound implied by  $X_1 \geq 0$

Measuring Noise Parameters

- Many different methods [20,22,24,26-37], most based on IEEE parameterization.
- Basic idea of (almost) all methods is to
  - present amplifier (or device) with a variety of different known input terminations ( $\Gamma$  &  $T$ ),
  - have an equation for the “output” in terms of the noise parameters and known quantities ( $\Gamma$ 's,  $T$ 's, S-parameters),
  - determine noise parameters by a fit to the measured output.
  - Need good distrib. of  $\Gamma$ 's in complex plane.

- Can fit for noise figure [24]

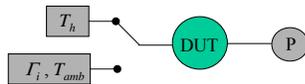


$$NF = NF_{\min} + \frac{4R_n}{Z_0} \frac{|\Gamma_{opt} - \Gamma_1|^2}{|1 + \Gamma_{opt}|^2 (1 - |\Gamma_1|^2)}$$

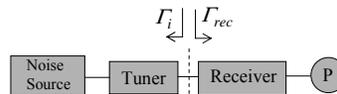
Notes:

- Use tuner to get different  $\Gamma_1$ , measure with  $T_h$  and  $T_c$  for each  $\Gamma_1$  to get NF for that  $\Gamma_1$ .
- Must correct for tuner to get  $T_{in}$  at 1. Must calibrate receiver for each value of  $\Gamma_2$  (or have isolator in front of receiver).

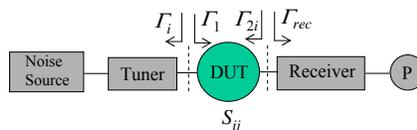
- Or can fit for output power [27, 38, 39]. This is the most popular method now.



In practice, first measure noise parameters of receiver,

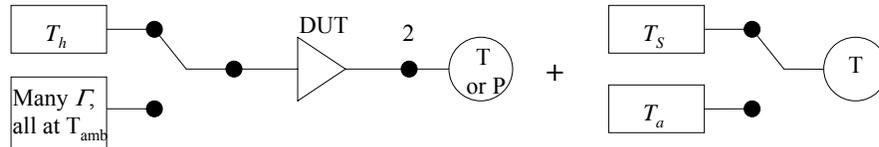


Then measure DUT + receiver



and extract DUT noise parameters.

- Noise-matrix approach [24,25,34,40] to measuring noise parameters:



$$k_B T_2 = \frac{|S_{21}|^2}{(1 - |\Gamma_{GS}|^2)} [N_G + N_1 + N_2 + N_{12}]$$

$$\Gamma_{GS} = S_{22} + \frac{\Gamma_G S_{12} S_{21}}{(1 - \Gamma_G S_{11})}$$

$$N_G = \frac{(1 - |\Gamma_G|^2)}{|1 - \Gamma_G S_{11}|^2} k_B T_G$$

$$N_1 = \left| \frac{\Gamma_G}{1 - \Gamma_G S_{11}} \right|^2 k_B X_1$$

$$N_2 = k_B X_2$$

$$N_{12} = 2 \operatorname{Re} \left[ \frac{\Gamma_G}{(1 - \Gamma_G S_{11})} k_B X_{12} \right]$$

- Noise-Parameter Uncertainties

- Monte Carlo method is probably the most practical [29,40 – 43]
- Some general approximate features [40]:
  - Uncerts in  $G$  and  $T_{\min}$  (&  $F_{\min}$ ) are dominated by uncert in  $T_h$ . 0.1 dB uncert in  $T_h \rightarrow \sim 0.1$  dB uncert in  $G$  and  $F_{\min}$ .
  - Uncerts in  $\Gamma_{opt}$  are dominated by uncerts in  $\Gamma_G$ 's. Uncert in Re or Im  $\Gamma_{opt}$  is  $\sim 3$  or  $4\times$  uncert in Re or Im  $\Gamma_G$  (for 13 terminations).
  - $t$  is sensitive to just about everything.
  - $T_{amb}$  is not a major factor, because it is known much better than  $T_h$ . Note, however, that it could affect  $T_h$  or the amplifier properties.

## Measuring Noise Parameters on Wafer

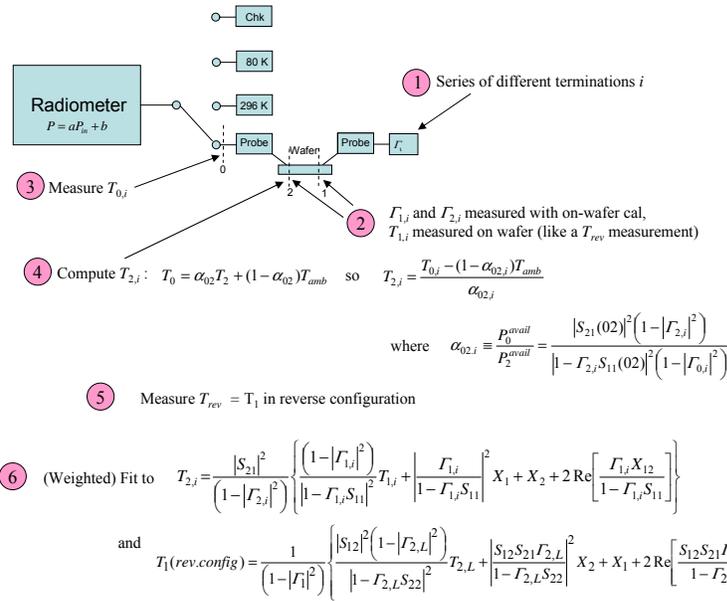
- Just like amplifier noise parameters—only harder.
- Harder due to probes and to device properties.
- Complications due to Probes:
  - Must characterize probes: on-wafer standards  
 $\Rightarrow$  larger uncertainties for  $\Gamma$ 's,  $S$ -parameters,  $T_{in}$ ,  $T_{out}$ .
  - Restricted range of  $\Gamma$ 's (due to loss in probe).
  - Potential contact problems, vibrations.

- Complications due to Device:
  - If measuring an on-wafer amplifier, no additional device-related problems.

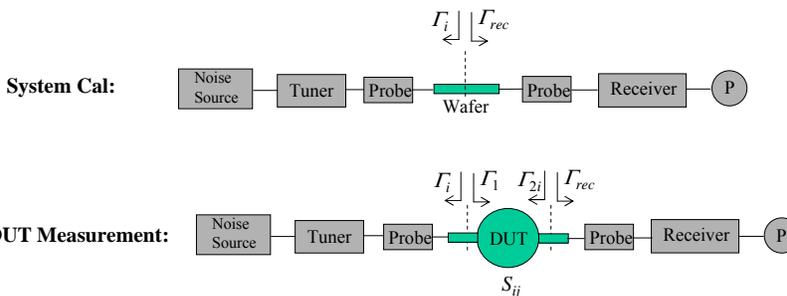
**But for a transistor:**

  - Matching problems, large  $S_{11}$ ,  $S_{22} \Rightarrow$  larger corrections & therefore larger uncertainties.
  - Large  $\Gamma_{opt}$ , near edge of Smith chart.
  - Smaller noise figures/noise temps than amps.

• Procedure used at NIST [44, 45]:

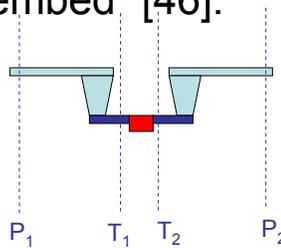


- Commercial Systems [e.g., 38, 39]: similar to general noise parameters (above), except that reference planes are on wafer.



- Must therefore calibrate at those reference planes on wafer. Commonly done at probe tip, with an “off-wafer” cal set.

- To get properties of device itself, must remove effects of lines between the calibration reference planes ( $P_1$  and  $P_2$ ) and the device reference planes ( $T_1$  and  $T_2$ ). To do so, measure auxiliary standards (short, open) between planes  $T_1$  and  $T_2$ , and “deembed” [46].



#### IV. NOISE-PARAMETER CHECKS & VERIFICATION

- So how do we convince ourselves that our noise-parameter measurement results might be correct?
- Will give three tests:
  - measure noise parameters of passive device, such as attenuator
  - measure  $T_{rev}$
  - measure noise parameters of isolator+amp

## Attenuator Test

- Noise matrix of a passive device (such as an attenuator) is given by Bosma's theorem,

$$\langle \hat{b}_i \hat{b}_j^* \rangle = kT (\mathbf{I} - \mathbf{S}\mathbf{S}^+)_{ij}$$

- So for an attenuator at (noise) temperature  $T_a$ ,

$$X_1 = (1 - |S_{11}|^2 - |S_{12}|^2) T_a$$

$$X_2 = \frac{(1 - |S_{22}|^2 - |S_{21}|^2)}{|S_{21}|^2} T_a$$

$$X_{12} = -\frac{(S_{21}^* S_{11} + S_{12} S_{22}^*)}{S_{21}^*} T_a$$

- So, measure noise parameters of an attenuator & see if you get the correct answers.

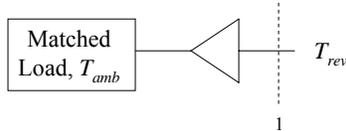
- Other passive devices as tests (especially on a wafer):

- Cold FET [47]
- Lange Coupler [48]

These have the advantage of being poorly matched, & therefore more similar to the devices of interest.

[T<sub>rev</sub> Test \[30,34,49\]](#)

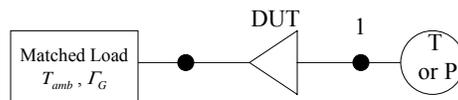
- $T_{rev}$  test: Measure noise temp from input of amplifier, when output is terminated in a matched load.



- Can show that for  $\Gamma_L S_{21} S_{12}$  small,

$$T_{rev} \approx \frac{X_1}{(1 - |\Gamma_1|^2)}$$

- Full form is:



$$T_1 = \frac{1}{(1 - |\Gamma_1|)} [N_G + N_1 + N_2 + N_{12}]$$

$$N_G = \frac{|S_{12}|^2 (1 - |\Gamma_G|^2)}{|1 - \Gamma_G S_{22}|^2} k_B T_{amb}$$

$$N_1 = k_B X_1$$

$$N_2 = \frac{|S_{12} S_{21} \Gamma_G|^2}{|1 - \Gamma_G S_{22}|^2} k_B X_2$$

$$N_{12} = 2 \operatorname{Re} \left[ \frac{S_{12} S_{21} \Gamma_G}{(1 - \Gamma_G S_{22})} k_B X_{12}^* \right]$$

$$\Gamma_1 = S_{11} + \frac{\Gamma_G S_{12} S_{21}}{(1 - \Gamma_G S_{22})}$$

- So measure  $T_{rev}$ , compare to value predicted from the value of  $X_1$  from the noise-parameter determination.
- If working in terms of IEEE parameters, convert, using

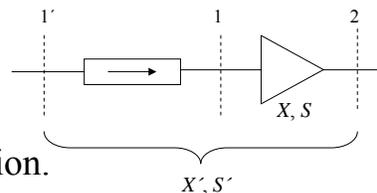
$$X_1 = T_{e,\min} (|S_{11}|^2 - 1) + \frac{t|1 - S_{11}\Gamma_{opt}|^2}{|1 + \Gamma_{opt}|^2},$$

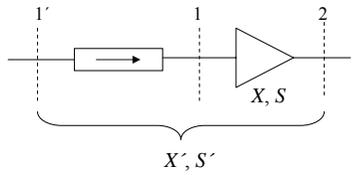
$$X_2 = T_{e,\min} + \frac{t|\Gamma_{opt}|^2}{|1 + \Gamma_{opt}|^2},$$

$$X_{12} = S_{11}T_{e,\min} - \frac{t\Gamma_{opt}^*(1 - S_{11}\Gamma_{opt})}{|1 + \Gamma_{opt}|^2}.$$

### “Isolator” Test [49]

- Connect isolator to amplifier input & measure noise parameters of combination.
- $X'$  parameters can be written in terms of  $X$  parameters (amp alone) and the  $S$ -parameters of amp and isolator.
- Using Bosma’s theorem and standard  $S$ -parameter algebra, can show





$$X_1' = \left| \frac{S_{12}'}{1 - S_{11}'S_{22}'} \right|^2 X_1 + T_i(A_1 - A_2),$$

$$A_1 = \left\{ \left( 1 - |S_{11}'|^2 - |S_{12}'|^2 \right) + \left| \frac{S_{11}'S_{12}'}{1 - S_{11}'S_{22}'} \right|^2 \left( 1 - |S_{21}'|^2 - |S_{22}'|^2 \right) \right\},$$

$$A_2 = 2 \operatorname{Re} \left[ \frac{S_{12}'S_{11}'}{(1 - S_{11}'S_{22}')} (S_{21}'S_{11}^{*'} + S_{12}'^{*'}S_{22}') \right],$$

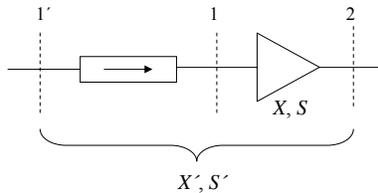
$$X_2' = \frac{1}{|S_{21}'|^2} \left\{ |1 - S_{11}'S_{22}'|^2 X_2 + |S_{22}'|^2 X_1 + 2 \operatorname{Re} \left[ S_{22}'(1 - S_{11}'S_{22}')^* X_{12} \right] + T_i \left( 1 - |S_{22}'|^2 - |S_{21}'|^2 \right) \right\},$$

$$X_{12}' = \frac{S_{12}'(1 - S_{11}'S_{22}')^*}{S_{21}'^{*'}(1 - S_{11}'S_{22}')} X_{12} + \frac{S_{12}'S_{22}'^{*'}}{S_{21}'^{*'}(1 - S_{11}'S_{22}')} X_1 - T_i A_3,$$

$$A_3 = \left[ \left( \frac{S_{21}'^{*'}S_{11}' + S_{12}'^{*'}S_{22}'}{S_{21}'^{*'}} \right) - \frac{S_{12}'S_{11}'}{S_{21}'^{*'}(1 - S_{11}'S_{22}')} \left( 1 - |S_{22}'|^2 - |S_{21}'|^2 \right) \right],$$

Note: could instead use an attenuator (for on wafer).

- Approximate expressions:



$$X_1' \approx T_i,$$

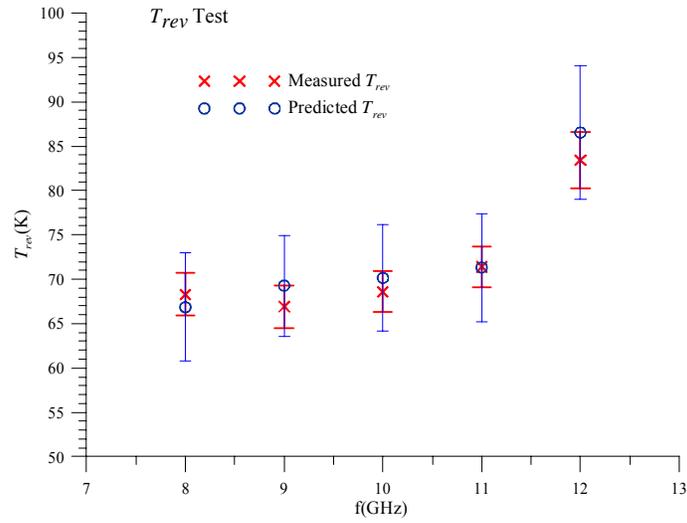
$$X_2' \approx \frac{\left( X_2 + T_i \left( 1 - |S_{21}'|^2 \right) \right)}{|S_{21}'|^2},$$

$$X_{12}' \approx \frac{S_{12}'}{S_{21}'^{*'}} X_{12} - T_i S_{11}',$$

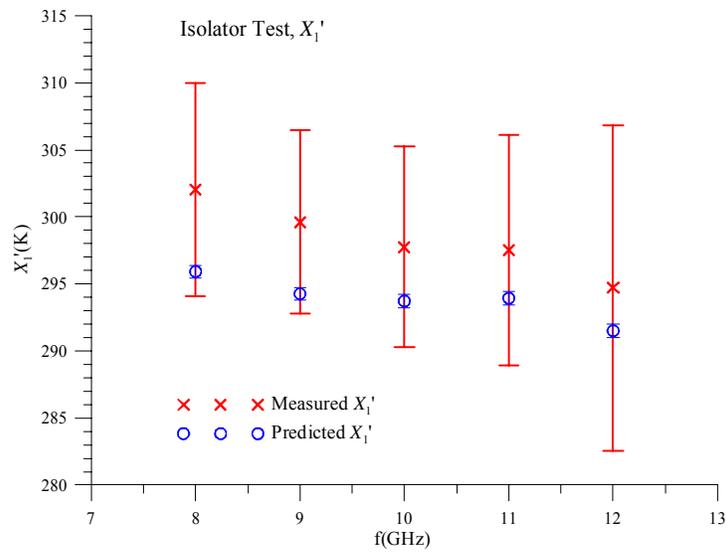
$X_{12}'$  is small and (approximately) independent of amplifier; excellent verification test.

## Samples of Test Results [49]

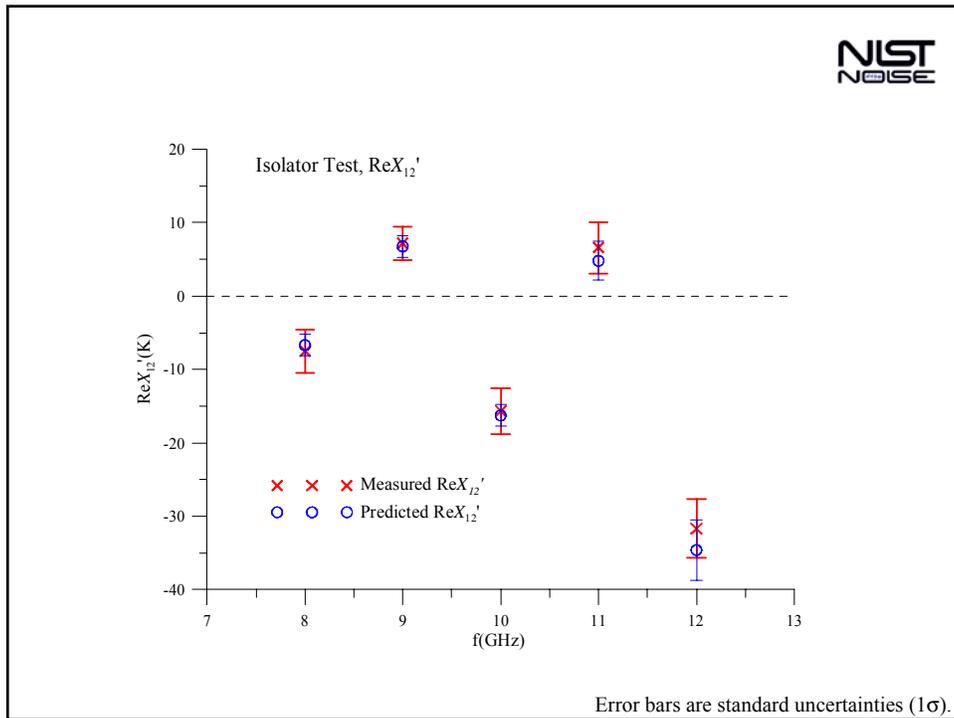
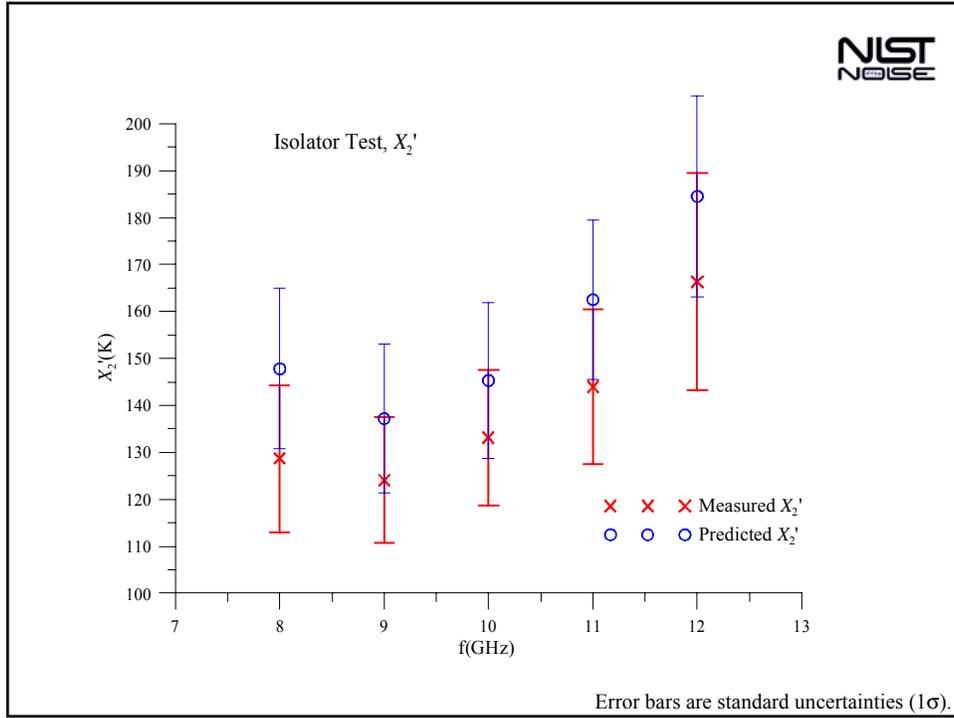
NIST  
NOISE

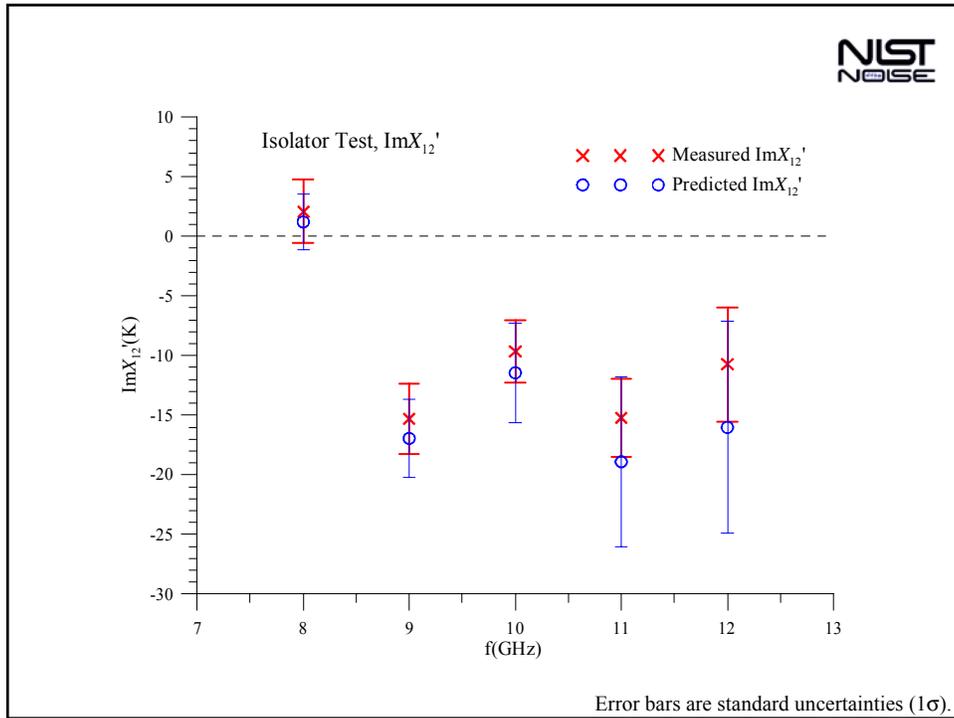


Error bars are standard uncertainties ( $1\sigma$ ).



Error bars are standard uncertainties ( $1\sigma$ ).





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