MULTIPORT NOISE CHARACTERIZATION AND DIFFERENTIAL AMPLIFIERS

J. Randa
RF Technology Division
NIST

55th ARFTG Conference, Boston, 6/ 16/00

OUTLINE

- # Background & Introduction
- # Formalism & Noise Matrix
- # Definition of Noise Figure for Multiports
- # Example 1: Differential amplifier, $'_i = 0$.
- # (Example 2: Four ports, $'_i = 0$.)
- # Summary

BACKGROUND & INTRODUCTION

- Noise figure is a measure of how much noise an amplifier or transistor adds to an input signal.
- Usual (IEEE, two port) definition: noise figure (or factor) at a given frequency = (total output noise per unit BW)/(portion due to input noise), evaluated for input noise = $k_B T_0$, where T_0 = 290 K.
- Noise figure not an intrinsic property; it depends on impedance or reflection coefficient of the source.
- Can parameterize dependence so that given Γ_{source} can calculate noise figure.

Several different parameterizations:

- IEEE (& variants):

$$F = F_{\min} + \frac{R_n}{G_s}|Y_s - Y_{opt}|^2$$

- Noise Matrix (see below):

$$\hat{N} = \begin{pmatrix} \overline{|\hat{b}_1|^2} & \overline{\hat{b}_1 \hat{b}_2^*} \\ \overline{\hat{b}_2 \hat{b}_1^*} & \overline{|\hat{b}_1|^2} \end{pmatrix}$$

- & others

That's nice, but what about multiple input and/or output ports?

1) How do define noise figure?

2) How do we parameterize it?

Why would we care? Differential amplifiers in cell phones & elsewhere

---

Formalism & Noise Matrix

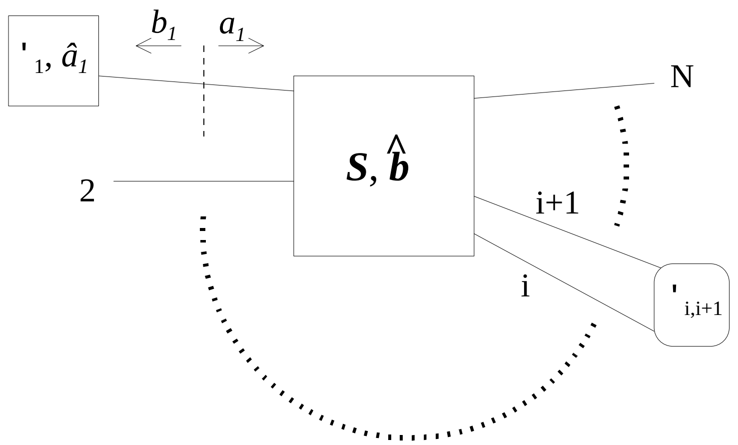
Port: either separate physical ports or separate modes; 2 modes in 1 physical port ° 2 "ports"

Use wave amplitudes. Assume

<Power-orthogonal

<Physically realizable

- Notation: N-port device, bold indicates vector or matrix.



$$\mathbf{a} = \mathbf{\Gamma} \mathbf{b} + \hat{\mathbf{a}}$$

$$\mathbf{b} = \mathbf{S} \mathbf{a} + \hat{\mathbf{b}}$$

$$\mathbf{b} = [\mathbf{1} - \mathbf{S}\mathbf{\Gamma}]^{-1}[\mathbf{S}\hat{\mathbf{a}} + \hat{\mathbf{b}}]$$

- Noise Matrix

$$\mathbf{N} \equiv \overline{\mathbf{b}\,\mathbf{b}^{\dagger}} \quad \text{or} \quad N_{ij} = \overline{b_i b_j^{*}}$$

$$\mathbf{N} = \overline{\mathbf{b}\,\mathbf{b}^{\dagger}} = [\mathbf{1} - \mathbf{S}\mathbf{\Gamma}]^{-1}\mathbf{S}\,\overline{\hat{\mathbf{a}}\hat{\mathbf{a}}^{\dagger}}\,\mathbf{S}^{\dagger}[\mathbf{1} - \mathbf{S}\mathbf{\Gamma}]^{-1\dagger}$$

$$+ [\mathbf{1} - \mathbf{S}\mathbf{\Gamma}]^{-1}\overline{\hat{\mathbf{b}}\hat{\mathbf{b}}^{\dagger}}[\mathbf{1} - \mathbf{S}\mathbf{\Gamma}]^{-1\dagger}$$

  - No external noise $\Rightarrow$ $\hat{a} = 0$
  - No internal amplifier noise $\Rightarrow$ $\hat{b} = 0$
  - Comment about contributions to n.f.

- Intrinsic Noise Matrix $\hat{\mathbf{N}} = \overline{\hat{\mathbf{b}}\,\hat{\mathbf{b}}^{\dagger}}$

Intrinsic noise matrix contains all the noise information of the N-port.

To make $\hat{N}$ more physical, let

$$\overline{|\hat{b}_i|^2} = k_B\,\hat{T}_i \qquad \overline{\hat{b}_i\,\hat{b}_j^*} = k_B\sqrt{\hat{T}_i\,\hat{T}_j}\;\Delta_{ij}$$

$\hat{T}_i$ *s are characteristic noise temperatures, & D_{ij} *s are correlation functions.

Then

$$\hat{N} = \begin{pmatrix} \hat{T}_1 & \sqrt{\hat{T}_1\,\hat{T}_2}\;\Delta_{12} & \sqrt{\hat{T}_1\,\hat{T}_3}\;\Delta_{13} & \ldots \\ \sqrt{\hat{T}_2\,\hat{T}_1}\;\Delta_{12}^* & \hat{T}_2 & \sqrt{\hat{T}_2\,\hat{T}_3}\;\Delta_{23} & \ldots \\ \sqrt{\hat{T}_3\,\hat{T}_1}\;\Delta_{13}^* & \sqrt{\hat{T}_3\,\hat{T}_2}\;\Delta_{23}^* & \hat{T}_3 & \ldots \\ \ldots & & & \end{pmatrix}$$

Reflectionless loads °

$$T_{i,0} = \frac{\hat{T}_i}{1-|S_{ii}|^2}$$

(Proposed) Multiport Noise Figure Definition

- Consider only NF at a single frequency (spot NF).
- IEEE standard does not define NF for multiple input (or output) ports.
- Definition for two-port was given above.

- To generalize that, for each (output) port, define the noise figure to be the ratio of the total output noise in that port to the output noise power which is due to the input noise for the case when the input noise is T_0 .

HOWEVER,

<How are all the ports terminated?

<T_0 to *all* input ports? Correlated?

<The portion due to *what* input noise? All? Just one input port?

<T_0 to other *output* ports?

<For differential amplifier, noise input to physical ports 1 & 2, or to differential & common modes? If to differential & common modes, how do you do that?

The configuration most like practical conditions would be T_0 input to *all* input ports. Also, T_0 *s at physically distinct input ports should be uncorrelated (fortunately).

Note: What about diff. mode, correlated? “No.”

Port 1: a_1 Port 2: a_2

$$a_{\pm} \text{ ' } (a_1 \pm a_2)/\sqrt{2}\,,$$

$$\overline{a_{\%}\, a_{!}^{(}} \text{ ' } (\overline{*a_1*^2} \;!\; \overline{*a_2*^2})/2 \;\%\; (T_1 \;!\; T_2) \;.\; 0$$

Output ports: no noise input (like 2-port case); should make little difference in practice.

Terminations: same '*s as actual configuration.

That specifies definition. It reduces to usual 2-port definition, & it serves as a measure of how much noise the amplifier adds to the input signal (but see below re: S/N).

In terms of the intrinsic noise matrix,

$$F_i \; ' \; 1 \; \% \; \frac{\{[\mathbf{1} \,!\, \boldsymbol{S}\,']^{\&1}\, \hat{N}\, [\mathbf{1} \,!\, \boldsymbol{S}\,']^{\&1\dagger}\}_{ii}}{\{[\mathbf{1} \,!\, \boldsymbol{S}\,']^{\&1}\, \boldsymbol{S}\, \overline{\hat{\boldsymbol{a}}\hat{\boldsymbol{a}}^{\dagger}(\boldsymbol{T}_0)}\, \boldsymbol{S}^{\dagger}\, [\mathbf{1} \,!\, \boldsymbol{S}\,']^{\&1\dagger}\}_{ii}}$$

$$\overline{\hat{\boldsymbol{a}}\hat{\boldsymbol{a}}^{\dagger}(\boldsymbol{T}_0)} \; ' \; (k_B\, T_0)\times\mathbf{1} \;\; \text{(for input ports only)}$$

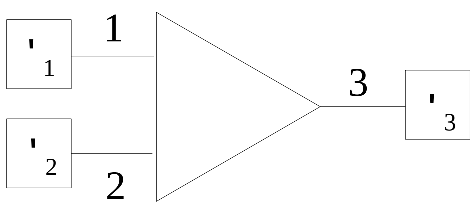
And that constitutes a parameterization of the multiport noise figure. Parameters are independent elements of the intrinsic noise matrix.

It may seem a bit formal, but it should become clearer in the example(s).

Note: an IEEE-type parameterization for 2÷1 port (diff. amp) would also require 9 parameters.

Example: Differential Amplifier, All '*s =0

Input ports 1 & 2, output port 3. Ideally, b_3 % (a_1 ! a_2).



$a_{\pm}$ / $(a_1 \pm a_2)/\sqrt{2}$

$S_{3\pm}$ ' $(S_{31} \pm S_{32})/\sqrt{2}$

'$_1$ ' '$_2$ ' '$_3$ ' 0

b_3 ' $S_{3!}a_!$ % $S_{3\%}a_\%$ % $\hat{b}_3$

- Output noise power per unit BW at port 3 is given by

$$N_3 = \overline{|S_{31}a_1 + S_{32}a_2 + \hat{b}_3|^2}$$

- If uncorrelated noise sources $T_1^{(i)}$ and $T_2^{(i)}$ are input, then

$$N_3 = G_{31}T_1^{(i)} + G_{32}T_2^{(i)} + \hat{T}_3$$

$$G_{31} = |S_{31}|^2, \quad G_{32} = |S_{32}|^2$$

- So to determine $\hat{T}_3$ & gains, measure with different $T_1^{(i)}$ and $T_2^{(i)}$ *s.

- Assume a hot & cold source for each input port: T_{h1}, T_{c1}, T_{h2}, T_{c2} .
- Let $N_{3,hc}$ be the output noise power at port 3 for the hot source on port 1 & the cold source on port 2, etc. Then (ignoring k_B)

$$\begin{aligned}
N_{3,hh} &= G_{31}T_{h1} + G_{32}T_{h2} + \hat{T}_3, \\
N_{3,hc} &= G_{31}T_{h1} + G_{32}T_{c2} + \hat{T}_3, \\
N_{3,ch} &= G_{31}T_{c1} + G_{32}T_{h2} + \hat{T}_3, \\
N_{3,cc} &= G_{31}T_{c1} + G_{32}T_{c2} + \hat{T}_3.
\end{aligned}$$

Four equations, three unkowns:

$$N_{3,hh} + N_{3,cc} = N_{3,hc} + N_{3,ch}$$

Say we measure hc, ch, cc; solve & get

$$\hat{T}_3 = \frac{(T_{h1}T_{h2} - T_{c1}T_{c2})}{(T_{h1} - T_{c1})(T_{h2} - T_{c2})} N_{3,cc} - \frac{T_{c1}}{(T_{h1} - T_{c1})} N_{3,hc} - \frac{T_{c2}}{(T_{h2} - T_{c2})} N_{3,ch}$$

$$G_{31} = \frac{N_{3,hc} - N_{3,cc}}{T_{h1} - T_{c1}}$$

$$G_{32} = \frac{N_{3,ch} - N_{3,cc}}{T_{h2} - T_{c2}}$$

Can define an effective input noise temperature (same for both input ports in this case)

$$T_e \equiv \frac{\hat{T}_3}{G_{31} + G_{32}}$$

Assume $T_{c1} = T_{c2}$, & let $Y_{ch} = N_{ch}/N_{cc}$, etc. Then

$$T_e = \frac{T_{h1}T_{h2} - (T_{h1}Y_{ch} + T_{h2}Y_{hc})T_c + Y_{hh}T_c^2}{(Y_{ch} - 1)T_{h1} + (Y_{hc} - 1)T_{h2} - (Y_{hh} - 1)T_c}$$

- If Y_{hh} not measured, use
  $Y_{hh} = Y_{hc} + Y_{ch} - 1.$
- If $T_{h1} = T_{h2}$,

$$T_e = \frac{T_h - Y_{hh} T_c}{Y_{hh} - 1}$$

$$G_{31} + G_{32} = \frac{N_{hh} - N_{cc}}{T_h - T_c}$$

$$\hat{T}_3 = \frac{(T_h - Y_{hh} T_c)(T_h - T_c)}{(Y_{hh} - 1)(N_{hh} - N_{cc})}$$

- So if $T_{h1} = T_{h2}$ and $T_{c1} = T_{c2}$, we can determine T_e and $\hat{T}_3$ from just 2 measurements.

- So what about differential & common mode?
- $\hat{T}_3$ is same whether choose ports 1,2,3 or +, −, 3.
- Also, can show $G_{3+} + G_{3-} = G_{31} + G_{32}$
- Since $\hat{T}_3$ and the sum of the gains are same whether use input ports 1,2 or +, −, effective input temperature T_e is the same for the common & differential modes as for ports 1 & 2.

Therefore, we can determine $\hat{T}_3$, T_e , and $(G_{3+} + G_{3!})$ for differential and common modes from the hot-cold measurements with uncorrelated sources on the physical ports 1 & 2.

More work required to get G_{3+} and $G_{3!}$ separately:

<Correlated input to ports 1 & 2

<Approximate $G_{3+} << G_{3!}$, so $G_{3!}$. $G_{3+} + G_{3!}$

<Measure some other way

So

<With just one hot source, can do hc, ch, cc to get $\hat{T}_3$, T_e , G_{31} , G_{32} , and $(G_{3+} + G_{3!})$.

<If have two equal-temperature hot sources, hh and cc suffice to determine $\hat{T}_3$, T_e , $(G_{31}+G_{32})$ and $(G_{3+} + G_{3!})$.

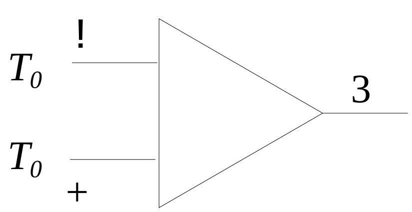
<To determine G_{3+} & $G_{3!}$ separately (in a noise measurement) requires input to ports 1 & 2 to be correlated.

- What about noise figure?
- From our definition,

$$F_3 = 1 + \frac{\overline{|\hat{b}_3|^2}}{|S_{31}|^2\,\overline{|\hat{a}_1|^2} + |S_{32}|^2\,\overline{|\hat{a}_2|^2}}$$

$$= 1 + \frac{\hat{T}_3}{(G_{31} + G_{32})\,T_0} = 1 + \frac{T_e}{T_0}.$$

- Complication: this noise figure does *not* measure the degradation of S/N.



$$G_{3-}\,T_0 + \hat{T}_3 + G_{3+}\,T_0$$

$$F(S/N) = \frac{(S/N)_{in}}{(S/N)_{out}} = \frac{N_{out}}{G_{3-}\,N_{in}}$$

$$F(S/N) = \frac{(G_{3-} + G_{3+})\,T_0 + \hat{T}_3}{G_{3-}\,T_0}$$

$$= \left(1 + \frac{G_{3+}}{G_{3-}}\right)\left(1 + \frac{T_e}{T_0}\right)$$

So F(S/N) differs from our definition of F by (1 + $G_{3+}/G_{3!}$). Must know G_{3+} and $G_{3!}$ separately to get F(S/N).

The two definitions differ in what they treat as the input noise. F(S/N) considers only the noise in the input (differential) channel.

Summary

Suggested a definition for noise figure of multiport (N$2) amplifiers.

Gave parameterization of that noise figure based on noise matrix, *i.e.*, expressed noise figure in terms of noise matrix.

Considered simple example in detail (2÷1 diff. amp, all ' ’s = 0).

Features of that example:

- Can input noise to physical ports; don’t need correlated (differential) noise input except to measure G_{3+} and $G_{3!}$ separately.
- 3 measurements (out of hh, hc, ch, cc) determine $\hat{T}_3$, T_e , F, G_{31} , G_{32} , and (G_{3+} + $G_{3!}$).
- If have two equal hot sources, hh and cc are enough for $\hat{T}_3$, T_e , F, G_{31}+G_{32} , and G_{3+}+$G_{3!}$
- Our F ... F(S/N) for multiport case.

Full paper also considers 4-port (2÷2) example with all ' ’s = 0. Interesting development there is that need different T_e for each input channel.

Future work will (may, should) address:

- ' ... 0 case & dependence upon all parameters in noise matrix.
- Some measurements employing this formalism.