Underlying Principles, Design, Construction, Test, and Uncertainties of the New NIST Coaxial Radiometer NFRad

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Outline

- Part I. Theory and Background
  - Probability & Statistical Mechanics background
  - Nyquist Theorem
  - Microwave networks and noise
  - Amplifiers & noise
  - Isolated total-power radiometer, radiometer equation
  - Unisolated total power radiometer

Part II. NFRad Design and Construction

- System overview, major components
- Important requirements and features
- Switch head
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- Switching and interfaces
- Electronics rack
- Plumbing

Part III. Operation, Testing, and Uncertainties

- Operation
- Software
- Testing
- Uncertainties
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PART I.

THEORY AND BACKGROUND

Outline

- Probability, statistical mechanics background
- Nyquist Theorem
- Microwave networks and noise
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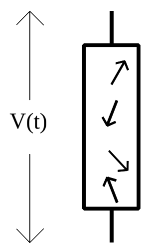
Probability, Statistical Mechanics Background [1]

- Thermal motion → fluctuating current & voltage



$I(t) = \Sigma i_i(t) = e\,\Sigma\, v_i(t)$

$V(t) = I(t)\,R$

$\langle I(t)\rangle = \langle V(t)\rangle = 0$

$\langle V^2\rangle, \langle I^2\rangle \propto T \neq 0$

$\langle P\rangle \propto T \neq 0$

- Fluctuating functions & probability
  - Ensemble & time averages; stationary, ergodic systems
  - (Auto) Correlation Function

Define autocorrelation function

$$K(s) \equiv \langle V(t')\,V(t'+s)\rangle_{equil}\,.$$

$K(0)$ is the dispersion (if $\langle V\rangle = 0$),

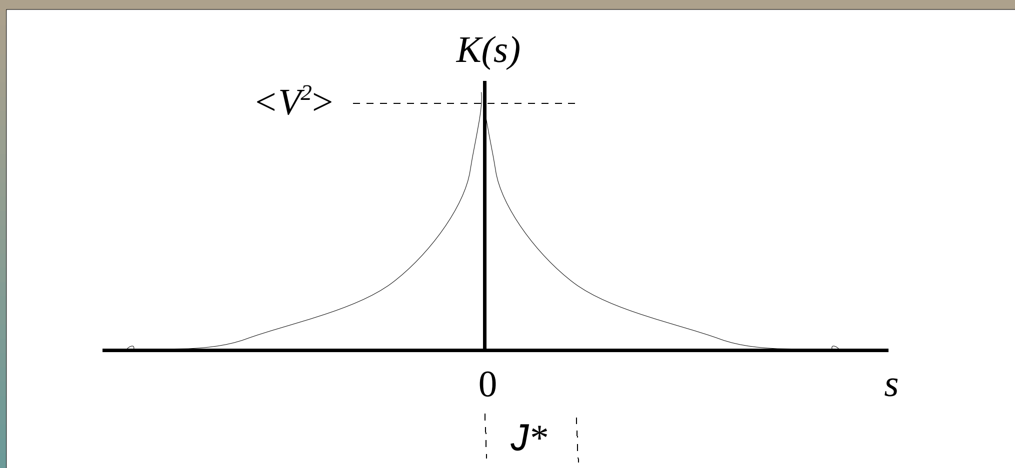
$$K(0) = \langle V^2(t)\rangle > 0\,.$$

$K(s)$ is independent of t; can show that

$$|K(s)| \leq K(0)\,; \quad K(s) = K(-s)\,.$$

#Fluctuating functions & probability

- < Correlation time *J**; *K(s)* 6 0 for *s* » *J**.
- < Correlation function looks roughly like



#Fourier Transform of fluctuating function:

- < FT not defined for random function defined over all time, so restrict function to !1 to 1 (1 very large),

$$v_1(t) \;/\; v(t), \quad !1 \# t \# 1$$
$$/ \;0 \quad otherwise.$$

Then

$$v_1(T) \;'\; \frac{1}{2B} \mathrm{m}_{\&4}^{4} v_1(t')\, e^{\&iTt'} dt',$$

$$v_1(t) \;'\; \mathrm{m}_{\&4}^{4} v_1(T)\, e^{iTt} dT .$$

Correlation Function & Wiener-Khintchine Relation

- Let $K(s)$ be the correlation function for v and define its FT $J(\omega)$,

$$K(s) \equiv \langle v_1(t)\, v_1(t+s) \rangle \,,$$

$$K(s) = \int_{-\infty}^{\infty} J(\omega)\, e^{-i\omega s}\, d\omega \,,$$

$$J(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} K(s)\, e^{-i\omega s}\, ds \,.$$

- These are Wiener-Khintchine Relations; to make them meaningful, must give physical meaning to $J(\omega)$.

Wiener-Khintchine (cont'd)

- Consider $K(0)$,

$$\langle v^2(t) \rangle = K(0) = \int_{-\infty}^{\infty} J(\omega)\, d\omega \,,$$

so that the average power dissipated in a resistance R is given by

$$\langle P \rangle = \frac{\langle v^2 \rangle}{R} = \frac{1}{R}\int_{-\infty}^{\infty} J(\omega)\, d\omega = \frac{1}{R}\int_{0}^{\infty} J_{+}(\omega)\, d\omega \,,$$

where $J_+(\omega) = J(\omega) + J(-\omega)$.

- So J_+ is the spectral density of v, or R times the power spectral density.

Wiener-Khintchine (cont'd)

- Also can relate J to v_1 by using

$$K(s) = \langle v_1(t)\, v_1(t+s)\rangle = \langle v_1(0)\, v_1(s)\rangle = \frac{1}{2T}\int_{-T}^{T} v(t')\, v(t'+s)\, dt'$$

in

$$J(\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} K(s)\, e^{-i\omega s}\, ds\,.$$

Regroup, recognize expressions for δ functions, to get

$$J(\omega) = \frac{\pi}{T}\, |v_1(\omega)|^2\,.$$

Nyquist Theorem [1]

Fluctuation-Dissipation Theorem:

- Usual (mechanical) treatment

$$m\frac{dv}{dt} = \Phi(t) + F(t)$$

$\Phi(t)$ is external force, slowly varying
$F(t)$ is internal, contains slow (F_0) and fast (F') variation

$$F(t) \approx -\gamma v + F'(t)\,, \qquad m\frac{dv}{dt} = \Phi(t) - \gamma v + F'(t)$$

γ is friction constant. Can show

$$\gamma = \frac{1}{2k_B T}\int \langle F(0)\, F(s)\rangle_{eq}\, ds$$

Fluctuation-Diss. Thm (cont'd)

- Electrical case, conductor with self inductance and resistance

$$L\frac{dI}{dt} = V_{ext}(t) + V(t) \approx V_{ext}(t) - RI + V'(t) \qquad (Langevin\ \ eqn)$$

Then

$$R = \frac{1}{2k_BT}\int_{-\infty}^{\infty} \langle V(0)V(s)\rangle_{eq}\, ds = \frac{1}{2k_BT}\, 2\pi\, J(0)$$

$$J_{+}(0) = \frac{2k_BTR}{\pi}$$

Nyquist Theorem

- $J(\omega)$ constant for $\omega\tau^* \ll 1$,

$$J(\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} K(s)\, e^{-i\omega s}\, ds$$

$K(s) \to 0$ for $s \gg \tau^*$, so $\omega \ll 1/\tau^* \Rightarrow \omega s \ll 1$. Therefore

$$J(\omega) \approx \frac{1}{2\pi}\int_{-\infty}^{\infty} K(s)\, 1\, ds = J(0)$$

So for $\omega \ll 1/\tau^*$

$$J_{+}(\omega) = \frac{2}{\pi} k_B T R$$

Nyquist Theorem (cont'd)

- So then

$$\langle P_{res}\rangle = \frac{\langle|V|^2\rangle}{R} = \frac{1}{R}\int_0^\infty J_\omega(\omega)\,d\omega = \frac{2k_BT}{\pi}\int d\omega$$

$$\frac{d\langle P_{avail}\rangle}{df} = \frac{1}{R}\langle|V/2|^2\rangle = k_BT,$$

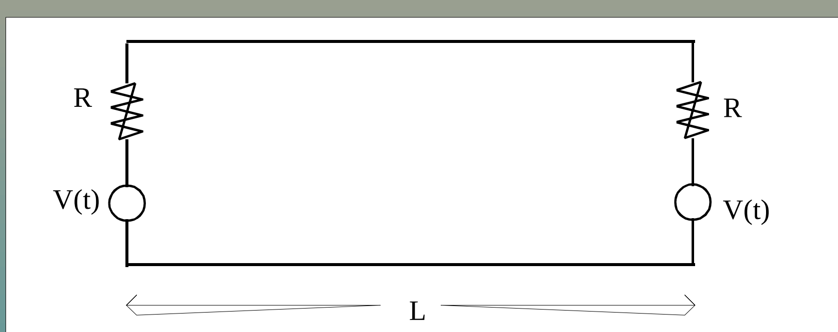
$$\langle P_{avail}(f)\rangle = k_BT\,df$$

- With quantum effects, it's really

$$\boxed{\langle P_{avail}(f)\rangle = \left(\frac{hf}{e^{\frac{hf}{k_BT}} - 1}\right) df}$$

Nyquist's derivation (with quantum update)

- Long transmission line, connecting identical R's
- Characteristic impedance R
- All in equilibrium at temperature T



Nyquist's derivation (cont'd)

- V(t) is the noise voltage in each resistor.
- One-dimensional black body problem.
- Voltage wave, $V = V_0 \exp[i(\omega t - kx)]$, velocity $c' = \omega k$.
- Count the number of modes:

Periodic boundary conditions: $V(x=L) = V(0)$, so $kL = 2\pi n$.

Number of modes per unit length is then $\Delta n = dk/(2\pi)$ between k and $k+\Delta k$, or between ω and $\omega + \Delta\omega$.

Nyquist's derivation (cont'd)

- From Planck distribution (B-E stats) the average number of photons in mode with frequency f_j is $\bar{n}_j = \dfrac{1}{e^{\epsilon_j/(k_B T)} - 1}$
- Energy of one photon in that mode is hf, so average energy in mode with f is

$$\langle \epsilon \rangle = \frac{hf}{e^{hf/(k_B T)} - 1}$$

- No reflections, so power incident on resistor is all absorbed.

Nyquist's derivation (cont'd)

- Thermal equilibrium, so power emitted = power absorbed (in each small frequency band ν to $\nu+d\nu$, detailed balance).
- Velocity×modes per length×energy per mode = energy per time absorbed/emitted,

$$dP_{em} = dP_{abs} = c\lambda \times \left(\frac{1}{2\pi}\frac{d\nu}{c\lambda}\right) \times \langle \varepsilon(\nu) \rangle = \frac{1}{2\pi}\left(\frac{hf}{e^{hf/(k_B T)} - 1}\right) d\nu$$

- The V from one resistor causes $I = V/(2R)$ to flow in the circuit, dissipating a power $P = R\langle I^2 \rangle = \langle V^2 \rangle/(4R)$ in the other resistor.

Nyquist's derivation (cont'd)

- That power must be the power emitted from the first resistor, so

$$\frac{\langle V^2 \rangle}{4R} = \frac{1}{4R}\int_0^\infty J_{\%}(\nu)\, d\nu = \frac{1}{2\pi}\int_0^\infty \left(\frac{hf}{e^{hf/(k_B T)} - 1}\right) d\nu$$

$$J_{\%}(\nu) = \frac{2}{\pi}\left(\frac{hf}{e^{hf/(k_B T)} - 1}\right) R$$

etc.

Quantum cutoff: f(GHz) $\approx$ 20 T(K)

In practice

- h/k_B = 0.04799 K/GHz

$$\langle P_{avail}(f)\rangle \approx \left(\frac{hf}{1 + \frac{hf}{k_B T} + \frac{1}{2}\left(\frac{hf}{k_B T}\right)^2 - 1} \right) df$$

$$\approx k_B T\left[1 - \frac{hf}{2\, k_B T}\right] df$$

- So at 290 K, 1 % effect at 116 GHz;
  at 100 K, 1 % effect at 40 GHz;
  at 100 K, 0.1 % effect at 4 Ghz.

Define: Noise Temperature (f) $\equiv$ [Available power spectral density]/k_B , so

$$\boxed{P_{avail} = k_B\, T_{noise}\, \Delta f}$$

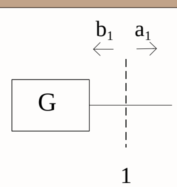
So for passive device

$$\boxed{T_{noise} = \frac{1}{k_B}\left(\frac{hf}{e^{\frac{hf}{k_B T}} - 1} \right)}$$

Microwave Networks & Noise [2,3]

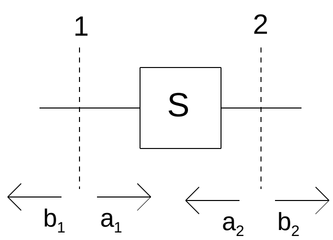
- Assume lossless lines, single mode.
- Travelling wave amplitudes *a*, *b*.
- Normalized such that $P_{del} = *a*^2 \,!\, *b*^2$
- May be a little careless about B; assume it's 1 Hz where needed.

- Describe (linear) one-ports by



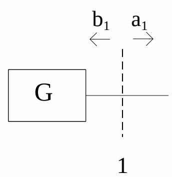
$$a_1 \;'\; '_G\, b_1 \,\%\, \hat{a}_G$$

- And (linear) two-ports by



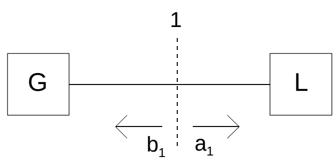
$$\begin{pmatrix} b_1 \\ b_2 \end{pmatrix} \;'\; \begin{pmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} \,\%\, \begin{pmatrix} \hat{b}_1 \\ \hat{b}_2 \end{pmatrix}$$

Available power:



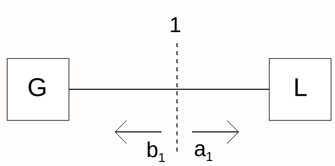
$$P_G^{(avail)} \; ' \; \frac{*\hat{a}_G*^2}{1 \; ! \; *'_G*^2} \qquad <*\hat{a}_G*^2> \; ' \; (1 \; ! \; *'_G*^2)\, k_B T_G$$

Delivered power:



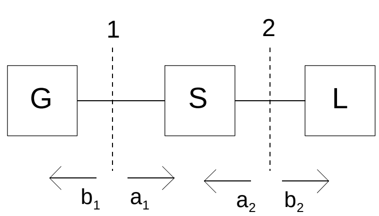
$$P_1^{del} \; ' \; *a_1*^2 \; ! \; *b_1*^2 \; ' \; *a_1*^2 (1 \; ! \; *'_L*^2)$$

Mismatch factor: M_1 / $p_{1,avail}/p_{1,del}$



$$M_1 \; ' \; \frac{(1 \; ! \; *'_L*^2)\,(1 \; ! \; *'_G*^2)}{*1 \; ! \; '_L \, '_G*^2}$$

Efficiency: 0_{21} / $p_{2,del}/p_{1,del}$

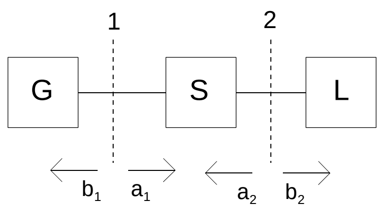


$$0_{21} \; ' \; \frac{*S_{21}*^2\,[1 \; ! \; *'_L*^2]}{*1 \; ! \; '_L S_{22}*^2\,[1 \; ! \; *'_{SL}*^2]}$$

$$0_{21} \; ' \; \frac{*S_{21}*^2\,[1 \; ! \; *'_L*^2]}{*1 \; ! \; S_{22}\,'_L*^2 \; ! \; *(S_{12}S_{21} \; ! \; S_{11}S_{22})\,'_L \; \% \; S_{11}*^2}$$

Available power ratio:

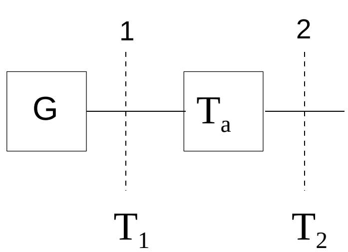
$$\eta_{21} \equiv p_{2,avail} / p_{1,avail} \; (\theta_1 , \theta_2 = 0)$$



$$\eta_{21} = \frac{|S_{21}|^2 [1 - |\Gamma_G|^2]}{|1 - \Gamma_G S_{11}|^2 [1 - |\Gamma_{GS}|^2]}$$

$$\Gamma_{GS} = S_{22} + \frac{S_{12} S_{21} \Gamma_G}{1 - \Gamma_G S_{11}}$$

For passive devices:



$$p_2^{avail} = \eta_{21} p_1^{avail} + f_0(T_a)$$

$$T_2 = \eta_{21} T_1 + f(T_a)$$

Say $T_1 = T_a$, then T_2 must $= T_a$, so

$$T_2 = T_a = \eta_{21} T_a + f(T_a)$$

$$f(T_a) = (1 - \eta_{21}) T_a$$

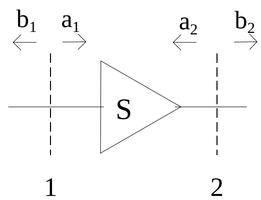
and therefore

$$\boxed{T_2 = \eta_{21} T_1 + (1 - \eta_{21}) T_a}$$

Practical: large vs. small T_1

Amplifiers & Noise [4–7]

#Linear two-port:

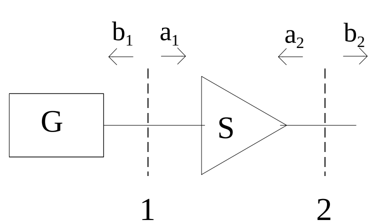


$$\begin{pmatrix} b_1 \\ b_2 \end{pmatrix} = \begin{pmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} + \begin{pmatrix} \hat{b}_1 \\ \hat{b}_2 \end{pmatrix}$$

#Four real noise parameters:

$$\langle |\hat{b}_1|^2 \rangle, \ \langle |\hat{b}_2|^2 \rangle, \ \langle \hat{b}_1 \hat{b}_2^* \rangle$$

#Output noise temperature T_2



$$k_B T_2 = \frac{|S_{21}|^2}{(1-|\Gamma_{GS}|^2)} [N_G + N_1 + N_2 + N_{12}]$$

$$N_G = \frac{(1-|\Gamma_G|^2)}{|1-\Gamma_G S_{11}|^2} T_G$$

$$N_1 = \left| \frac{\Gamma_G}{1-\Gamma_G S_{11}} \right|^2 \langle |\hat{b}_1|^2 \rangle$$

$$N_2 = \langle |\hat{b}_2 / S_{21}|^2 \rangle$$

$$N_{12} = 2 \, Re \left[\frac{\Gamma_G}{(1-\Gamma_G S_{11})} \langle \hat{b}_1 (\hat{b}_2 / S_{21})^* \rangle \right]$$

- Important point: T_2 depends on Γ_G. So same T_G can give different T_2, depending on Γ_G.
- IEEE parameterization [8–10] (actually a variation [4] of it):

$$T_e = T_{e,\min} + 4\,T_0 \frac{R_n}{Z_0} \frac{|\Gamma_G - \Gamma_{opt}|^2}{|1+\Gamma_{opt}|^2 (1 - |\Gamma_G|^2)}$$

$$T_2 = G\,(T_G + T_e)$$

- Noise figure is defined by

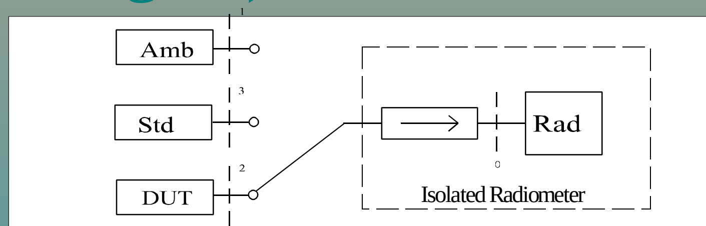
$$F(dB) = 10 \log_{10}\left(\frac{T_e + T_0}{T_0}\right)$$

$$T_0 = 290\ K$$

- Measure noise parameters by measuring T_2 for various T_G and Γ_G.
- Etc.

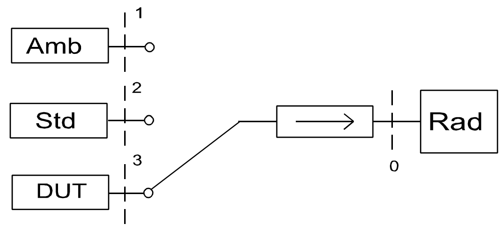
Isolated, Total-Power Radiometer [11,12]

- Basic idea: $P_{meter} = a + bP_{in}$
- Measure two standards; determine a (- system noise temperature) and b (system gain).



- System characteristics (a, b) must not change with source, so use isolators.
- P_{in} is minuscule, so amplify; LNA first. (100 K - 1.38 fW/MHz)
- Filter, downconvert to IF, amplify & filter some more.

Radiometer equation:



$$p_{0,x} \; ' \; M_{0,x}\,"_{03}P_x \,\%\, M_{0,x}(1 \,!\, "_{03})P_a \,\%\, p_{ex}$$

$$p_{0,S} \; ' \; M_{0,S}\,"_{02}P_S \,\%\, M_{0,S}(1 \,!\, "_{02})P_a \,\%\, p_{eS}$$

$$p_{0,a} \; ' \; M_{0,a}\,"_{01}P_a \,\%\, M_{0,a}(1 \,!\, "_{01})P_a \,\%\, p_{ea} \; ' \; M_{0,a}P_a \,\%\, p_{ea}$$

- Where P is available power & p is delivered power.

Radiometer equation (cont'd)

- Use $M_{0,x}"_{03} = M_3 O_{03}$, $M_{0,S}"_{02} = M_s O_{02}$ to get

$$p_{0,x} \; ' \; M_3\,0_{03}(P_x \,!\, P_a) \,\%\, M_{0,x}P_a \,\%\, p_{ex}$$

$$p_{0,S} \; ' \; M_S\,0_{02}(P_S \,!\, P_a) \,\%\, M_{0,S}P_a \,\%\, p_{eS}$$

$$p_{0,a} \; ' \; M_{0,a}P_a \,\%\, p_{ea}$$

- Because of the isolator, $M_{0,x} = M_{0,S} = M_{0,a}$ and $p_{ex} = p_{eS} = p_{ea}$, and so

$$p_{0,x} \; ' \; M_3\,0_{03}(P_x \,!\, P_a) \,\%\, p_{0,a}$$

$$p_{0,S} \; ' \; M_S\,0_{02}(P_S \,!\, P_a) \,\%\, p_{0,a}$$

- Radiometer equation (cont'd)
  - Rearrange a bit and divide one by the other to get

$$\frac{p_{0,x} - p_{0,a}}{p_{0,S} - p_{0,a}} \approx \frac{M_x \eta_{03} (P_x - P_a)}{M_S \eta_{02} (P_S - P_a)}$$

  - Use $P = k_B TB$ and rearrange some more to get the RADIOMETER EQUATION:

$$\boxed{T_x = T_a + \frac{M_S \eta_{02}}{M_x \eta_{03}} \frac{(Y_x - 1)}{(Y_S - 1)} (T_S - T_a)}$$

Unisolated, Total-Power Radiometer [13]

- Below about 1 GHz, isolators get large & cumbersome.
- Two options for total-power radiometer for low frequencies:
  - Tunable standards [14]
  - Characterize system as function of input Γ

- Tunable standards [14]: big disadvantage is that they are essentially single frequency.
- Characterize radiometer [13]: receiver noise temperature and gain vary with Γ_G same way amplier’s do. Measure two known noise temperatures and several different known impedances to determine radiometer parameters at each frequency.

Summary: Essential Results

- Noise temperature is defined by

$$\boxed{P_{avail} = k_B T_{noise} \Delta f}$$

- For a passive one-port device

$$\boxed{T_{noise} = \frac{1}{k_B}\left(\frac{hf}{e^{\frac{hf}{k_B T_{phys}}} - 1}\right) \approx T_{phys}}$$

For a passive two-port

$$T_2 = \alpha_{21} T_1 + (1 - \alpha_{21}) T_a$$

Radiometer equation for isolated, total-power radiometer

$$T_x = T_a + \frac{M_S \eta_{02}}{M_x \eta_{03}} \frac{(Y_x - 1)}{(Y_S - 1)} (T_S - T_a)$$