CHECKS OF AMPLIFIER NOISE-PARAMETER MEASUREMENTS

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I. INTRODUCTION

- NIST has developed capability to make accurate measurements of noise parameters of LNAs.
- Key component of any such capability is assessment of uncertainties.
- Need a method for checking that results (and uncertainties) are correct.
- Redundant measurements provide a consistency check (goodness of fit, small residuals).

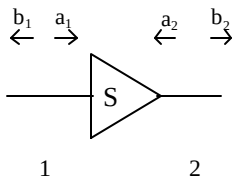
- Would like an amplifier with known noise parameters and/or a consistency check.
- A passive device (*e.g.*, an attenuator) has calculable noise parameters, but it tests limited range of possible errors.
- We suggest three different tests, and demonstrate two of them:
  - T_{rev} test,
  - Isolator (with amp) test,
  - Attenuator (with amp) test.

OUTLINE

- Formalism/framework
- Noise-Parameter Measurements
  - Method
  - Uncertainties
  - Results
- Checks & Verification
  - Theory
  - Measurements & Comparisons
- Discussion & Conclusions

II. FORMALISM/FRAMEWORK

- Will work with wave representation of noise matrix,



$$\begin{pmatrix} b_1 \\ b_2 \end{pmatrix} = \begin{pmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} + \begin{pmatrix} c_1 \\ c_2 \end{pmatrix}$$

$$X_1 \equiv \overline{|c_1|^2}$$

$$X_2 \equiv \overline{|c_2 / S_{21}|^2},$$

$$X_{12} \equiv \overline{c_1 c_2^* / S_{21}^*},$$

- Can relate the X's to the usual IEEE parameters,

$$T_e = T_{\min} + t \frac{|G_{opt} - G_G|^2}{|1 + G_{opt}|^2 (1 - |G_G|^2)}, \qquad t = \frac{4T_0 R_n}{Z_0}$$

$$T_e = \frac{|G_G|^2}{(1 - |G_G|^2)} X_1 + \frac{|1 - G_G S_{11}|^2}{(1 - |G_G|^2)} X_2 + \frac{2}{(1 - |G_G|^2)} \mathrm{Re}\left[(1 - G_G S_{11})^* G_G X_{12}\right]$$

- Relationship is generally somewhat messy; one simple case is $X_2 = T_{e,0}$.

- General relationships:

X's → IEEE

$$t = X_1 + |1+S_{11}|^2 X_2 - 2\,\mathrm{Re}\left[(1+S_{11})^* X_{12}\right],$$

$$T_{e,\min} = \frac{X_2 - |G_{opt}|^2\left[X_1 + |S_{11}|^2 X_2 - 2\,\mathrm{Re}\left(S_{11}^* X_{12}\right)\right]}{\left(1+|G_{opt}|^2\right)},$$

$$G_{opt} = \frac{h}{2}\left(1-\sqrt{1-\frac{4}{|h|^2}}\right),$$

$$h = \frac{X_2\left(1+|S_{11}|^2\right) + X_1 - 2\,\mathrm{Re}\left(S_{11}^* X_{12}\right)}{\left(X_2 S_{11} - X_{12}\right)}.$$

IEEE → X's

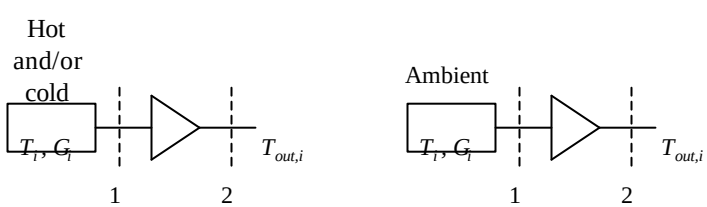
$$X_1 = T_{e,\min}\left(|S_{11}|^2 - 1\right) + \frac{t\,|1-S_{11}G_{opt}|^2}{|1+G_{opt}|^2},$$

$$X_2 = T_{e,\min} + \frac{t\,|G_{opt}|^2}{|1+G_{opt}|^2},$$

$$X_{12} = S_{11}T_{e,\min} - \frac{t\,G_{opt}^*\left(1-S_{11}G_{opt}\right)}{|1+G_{opt}|^2}.$$

II. NOISE-PARAMETER MEASUREMENTS

Method



$$T_{out} = T_{out}(G_i, T_i, S_{ij}, X_1, X_2, X_{12})$$

Measure for several (1 hot + 7 ambient) terminations and fit for X_1, X_2, X_{12} , and G_0.

$$G_0 \equiv \frac{|S_{21}|^2}{\left(1-|S_{11}|^2\right)}$$

Uncertainties

- Type A (statistical): obtained in the fitting process, $u_A(i) = \sqrt{Cov_{ii}}$
- Type B (other): from Monte Carlo program; input uncertainties in reflection coefficients, noise temperature of non-ambient source, ambient temperature, measurement of output noise temperature (or power), correlations, …
- Standard (combined): $u_c = \sqrt{u_A^2 + u_B^2}$

- Uncertainties in IEEE parameters:

$$u_i(IEEE) = \sqrt{V_{ii}(IEEE)}$$

$$V_{ij}(IEEE) = \sum_{i',j'=1}^{5} D_{ii'} D_{jj'} V_{i'j'}(X's)$$

$$D = \begin{pmatrix} \frac{\partial T_{\min}}{\partial X_1} & \frac{\partial T_{\min}}{\partial X_2} & \frac{\partial T_{\min}}{\partial \operatorname{Re} X_{12}} & \frac{\partial T_{\min}}{\partial \operatorname{Im} X_{12}} & 0 \\ \frac{\partial t}{\partial X_1} & \frac{\partial t}{\partial X_2} & \frac{\partial t}{\partial \operatorname{Re} X_{12}} & \frac{\partial t}{\partial \operatorname{Im} X_{12}} & 0 \\ \frac{\partial \operatorname{Re} G_{opt}}{\partial X_1} & \dots & \dots & \dots & 0 \\ \frac{\partial \operatorname{Im} G_{opt}}{\partial X_2} & \dots & \dots & \dots & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

Results

TABLE I
MEASURED VALUES OF THE NOISE PARAMETERS FOR THE AMPLIFIER ALONE.

	8 GHz	9 GHz	10 GHz	11 GHz	12 GHz
X_1(K)	64.5 ± 5.9	67.7 ± 5.5	68.6 ± 5.8	70.4 ± 6.0	80.3 ± 7.0
X_2(K)	110.0 ± 8.4	115.7 ± 7.8	117.6 ± 8.2	124.6 ± 8.4	134.4 ± 10.7
ReX_{12}(K)	9.4 ± 1.5	-7.2 ± 1.2	8.2 ± 1.2	-10.2 ± 1.0	14.3 ± 1.5
ImX_{12}(K)	20.2 ± 1.3	-14.3 ±1.4	10.7 ± 1.5	-14.3 ± 1.6	18.9 ± 2.8
G_0	2031 ± 32	2047 ± 30	1987 ± 30	2121 ± 33	1649 ± 31
T_{min}(K)	112.6 ± 8.4	112.2 ± 7.8	115.1 ± 8.2	123.4 ± 8.4	133.4 ± 10.6
t(K)	128.3 ± 2.6	234.2 ± 3.8	145.8 ± 3.0	223.9 ± 3.9	209.8 ± 4.5
ReG_{opt}	-0.172 ± 0.004	0.130 ± 0.004	-0.115 ± 0.005	0.077 ± 0.005	-0.006 ± 0.004
ImG_{opt}	0.101 ± 0.008	-0.046 ±. 0.009	-0.003 ± 0.008	-0.004 ± 0.008	-0.069 ± 0.008

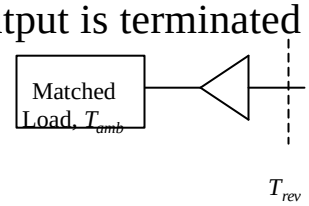
Max(c^2/DOF) = 0.32

Uncertainties are standard uncertainties (1σ).

III. TESTS & VERIFICATION

T_{rev} Test

- Measure noise temp from input of amplifier, when output is terminated in a matched load.



- Can show that for $G_L S_{21} S_{12}$ small,

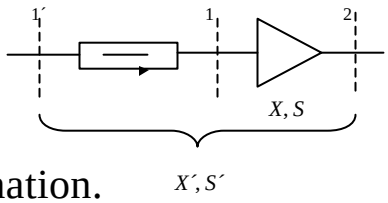
$$T_{rev} = \frac{X_1}{\left(1 - |G_1|^2\right)}$$

- So measure T_{rev}, compare to value predicted from the value of X_1 from the noise-parameter determination.
- If working in terms of IEEE parameters, use

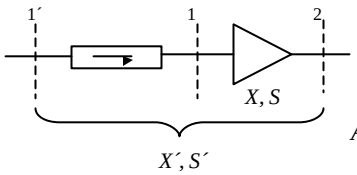
$$T_{rev} = \left[T_{e,\min}\left(\left|S_{11}\right|^2 - 1\right) + \frac{t\left|1 - S_{11}G_{opt}\right|^2}{\left|1 + G_{opt}\right|^2} \right]\left(1 - \left|G_1\right|^2\right)^{-1}$$

"Isolator" Test

- Connect isolator to amplifier input & measure noise parameters of combination.



- X' parameters can be written in terms of X parameters (amp alone) and the S-parameters of amp and isolator.
- Using Bosma's theorem and standard S-parameter algebra, can show



$$X_1' = \left|\frac{S_{12}^I}{1-S_{11}S_{22}^I}\right|^2 X_1 + k_B T_I (A_1 - A_2),$$

$$A_1 = \left\{\left(1-\left|S_{11}^I\right|^2-\left|S_{12}^I\right|^2\right)+\left|\frac{S_{11}S_{12}^I}{1-S_{11}S_{22}^I}\right|^2\left(1-\left|S_{21}^I\right|^2-\left|S_{22}^I\right|^2\right)\right\},$$

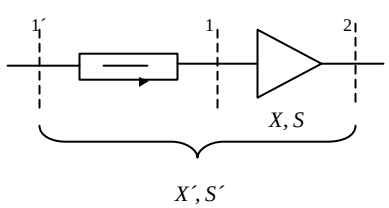
$$A_2 = 2\,\mathrm{Re}\left[\frac{S_{12}^I S_{11}}{(1-S_{11}S_{22}^I)}\left(S_{21}^I S_{11}^{I*} + S_{12}^{I*} S_{22}^I\right)\right],$$

$$X_2' = \frac{1}{\left|S_{21}^I\right|^2}\left\{\left|1-S_{11}S_{22}^I\right|^2 X_2 + \left|S_{22}^I\right|^2 X_1 + 2\,\mathrm{Re}\left[S_{22}^I\left(1-S_{11}S_{22}^I\right)^* X_{12}\right] + k_B T_I\left(1-\left|S_{22}^I\right|^2-\left|S_{21}^I\right|^2\right)\right\},$$

$$X_{12}' = \frac{S_{12}^I\left(1-S_{11}S_{22}^I\right)^*}{S_{21}^{I*}\left(1-S_{11}S_{22}^I\right)} X_{12} + \frac{S_{12}^I S_{22}^{I*}}{S_{21}^{I*}\left(1-S_{11}S_{22}^I\right)} X_1 - k_B T_I A_3,$$

$$A_3 = \left[\left(\frac{S_{21}^{I*}S_{11}^I + S_{12}^I S_{22}^{I*}}{S_{21}^{I*}}\right) - \frac{S_{12}^I S_{11}}{S_{21}^{I*}\left(1-S_{11}S_{22}^I\right)}\left(1-\left|S_{22}^I\right|^2-\left|S_{21}^I\right|^2\right)\right],$$

- Approximate expressions:



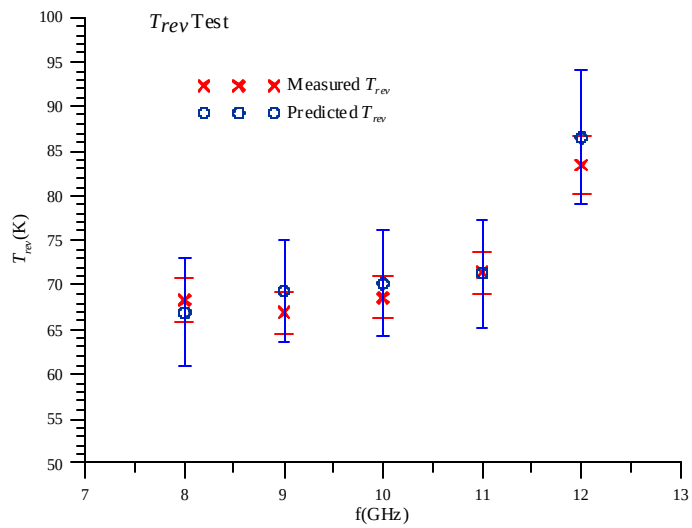
$$X_1' \approx k_B T_I,$$

$$X_2' \approx \frac{\left(X_2 + k_B T_I\left(1-\left|S_{21}^I\right|^2\right)\right)}{\left|S_{21}^I\right|^2},$$

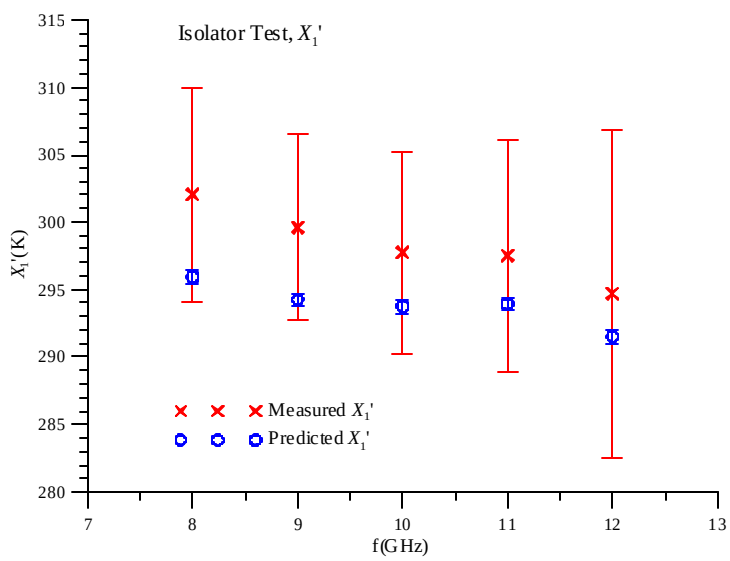
$$X_{12}' \approx \frac{S_{12}^I}{S_{21}^{I*}} X_{12} - k_B T_I S_{11}^I,$$

X_{12}' is small and (approximately) independent of amplifier; excellent verification test.

Results of Tests:



Error bars are standard uncertainties (1σ).



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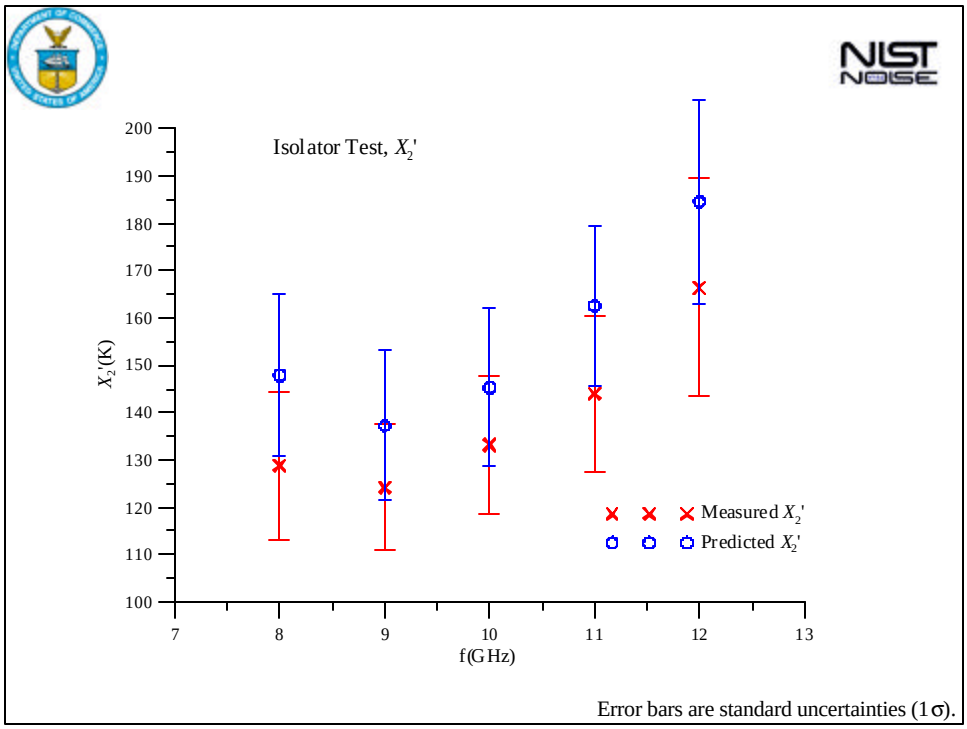
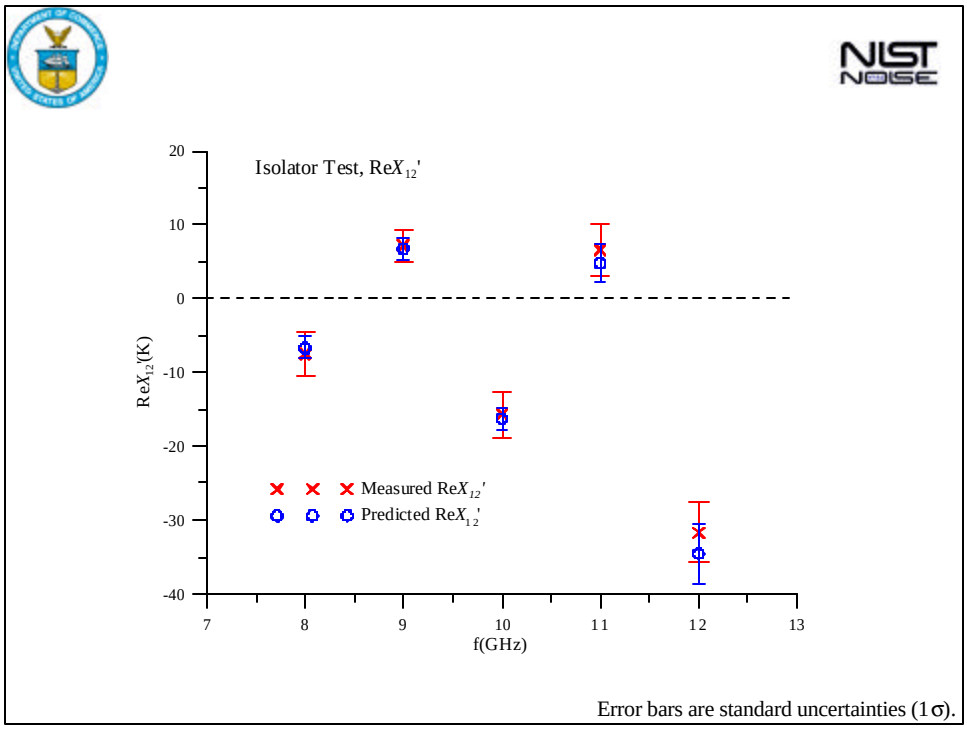
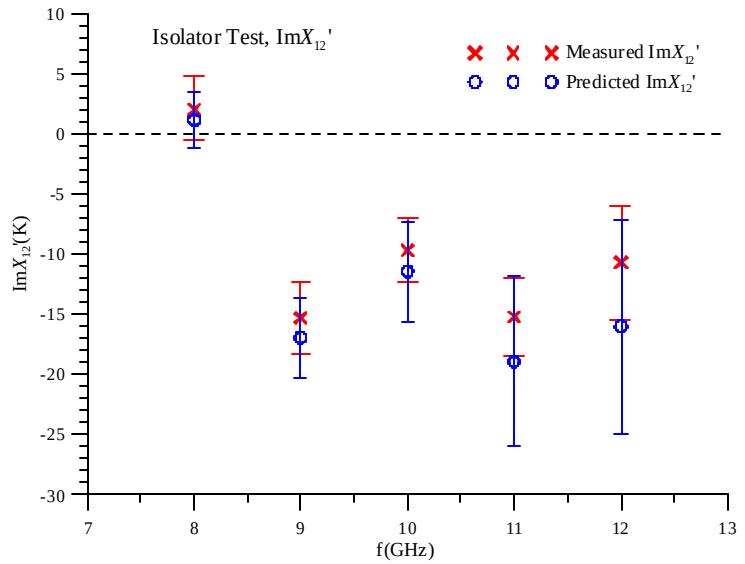

Isolator Test, X_2'
X_2'(K)
f(GHz)
Measured X_2'
Predicted X_2'
Error bars are standard uncertainties (1σ).


Isolator Test, ReX_{12}'
ReX_{12}'(K)
f(GHz)
Measured ReX_{12}'
Predicted ReX_{12}'
Error bars are standard uncertainties (1σ).



Error bars are standard uncertainties (1σ).

IV. SUMMARY

- NIST has developed capability to make accurate measurements of noise parameters of LNAs.
- Have developed and applied methods for checking noise-parameter measurements.
- Checks confirm both the results and the uncertainties of our measurements.
- Could use attenuator (~1 dB) instead of isolator in the checks. Would give more of a consistency check, as opposed to an absolute verification (X_{12}).

- Next:
  - →Reduce uncertainties.
  - →Comparison with NPL.
  - →U.S. round-robin or measurement service?
  - →Other frequencies (1 - 12.4 GHz).
  - – Checks with attenuators.
  - – Automate (variable termination unit, data collection & analysis software).
  - – On-wafer (attenuator check), ...

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