Series-Resistor Calibration

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Abstract- **We develop a coplanar-waveguide probe-tip scattering parameter calibration based on a thru, a reflect, and an accurately modeled series resistor. Comparison to a multiline Thru-Reflect-Line calibration verifies the accuracy of the method.**

I. INTRODUCTION

We develop a calibration based on a short transmission line (thru), a symmetric reflect, and a series resistor embedded in the line. We verify the accuracy of this series-resistor calibration in coplanar waveguide (CPW) by comparing it to an accurate multiline TRL calibration.

While the multiline thru-reflect-line (TRL) calibration [1] accurately calibrates wafer-probe stations, it requires long transmission line standards. Williams and Marks [2] introduced an accurate and compact line-reflect-match (LRM) calibration with verified accuracy based on characterized match and line standards. However, that LRM calibration requires identical match standards at the two ports; this may require wafer rotation during the calibration. It is also difficult to model accurately shunt resistors grounded with via holes.

In this paper we introduce a calibration that circumvents these problems. The calibration is based on a thru, a symmetric reflect, and an accurately modeled series resistor embedded in a short length of the line. The series-resistor model consists of simple, easily characterized lumped elements. A single measurement of the embedding circuit (a series-open CPW test pattern) without the resistor allows determination of all model elements except one; measurement of the dc resistance of the series-resistor standard determines the last element. This procedure eliminates the requirement of identical resistor standards on each port and in a microstrip environment also eliminates difficult-to-model grounding via holes.

II. CALIBRATION ALGORITHM

We developed a flexible scattering parameter calibration algorithm for two-port network analyzers to demonstrate the utility of series resistors as calibration standards. The algorithm uses a measurement of a thru and any combination of measurements of previously characterized one-port or two-port standards sufficient to determine the calibration constants of the network analyzer. The algorithm treats the thru standard as ideal and its measurements as perfect and finds a least-squares solution to the remaining undetermined calibration constants from the other standards: these standards set the calibration reference plane and reference impedance.

If we ignore switching and isolation errors, the relationship between the transmission matrix $\boldsymbol{M}_i$ of a two-port measured by the network analyzer and its actual transmission matrix $\boldsymbol{T}_i$ is

$$\boldsymbol{M}_i = \boldsymbol{X}\,\boldsymbol{T}_i\,\overline{\boldsymbol{Y}}, \tag{1}$$

where

$$\overline{\boldsymbol{Y}} = \boldsymbol{Q}\,\boldsymbol{Y}^{-1}\boldsymbol{Q}, \tag{2}$$

$$\boldsymbol{Q} \equiv \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \tag{3}$$

and $\boldsymbol{X}$ and $\boldsymbol{Y}$ are the "error boxes" describing the imperfections in the analyzer and its connections to the two-port. The calibration procedure determines $\boldsymbol{X}$ and $\boldsymbol{Y}$ from measurements $\boldsymbol{M}_i$ of standards with known transmission matrices $\boldsymbol{T}_i$. Once $\boldsymbol{X}$ and $\boldsymbol{Y}$ are known, equation (1) is easily inverted to determine the actual transmission matrix $\boldsymbol{T}$ of any circuit connected to the analyzer from its transmission matrix $\boldsymbol{M}$ measured by the analyzer.

We can set either $\boldsymbol{X}$ or $\boldsymbol{Y}$ reciprocal without losing any generality of the model. We chose $\boldsymbol{X}$ to be reciprocal, which allows us to write it as

$$\boldsymbol{X} = r\begin{bmatrix} 1 & a \\ b & c \end{bmatrix}, \tag{4}$$

where $r = (c-ab)^{-1/2}$: this eliminates one unknown in $\boldsymbol{X}$. We can use the fact that the transmission matrix $\boldsymbol{T}_{\mathrm{T}}$ of a thru line is the identity matrix to write $\boldsymbol{Y}$ as

$$\boldsymbol{Y} = \boldsymbol{Q}\,\boldsymbol{M}_{\mathrm{T}}^{-1}\,\boldsymbol{X}\,\boldsymbol{Q}, \tag{5}$$

where $\boldsymbol{M}_{\mathrm{T}}$ is the transmission matrix of the thru measured by the analyzer. This eliminates four more unknowns: only a, b, and c are still to be determined.

We use the additional measurements of one-port or two-port standards, or both, to determine a, b, and c with a least-squares algorithm. For two-port circuits, substituting (5) into (1) gives $\boldsymbol{M}_i\boldsymbol{M}_{\mathrm{T}}^{-1}\boldsymbol{X}=\boldsymbol{X}\,\boldsymbol{T}_i$, which imposes four conditions on the unknowns a, b, and c. These conditions are

$$-T_{i21}\,a + (\boldsymbol{M}_i\boldsymbol{M}_{\mathrm{T}}^{-1})_{12}\,b = T_{i11} - (\boldsymbol{M}_i\boldsymbol{M}_{\mathrm{T}}^{-1})_{11}, \tag{6}$$

$$[(\boldsymbol{M}_i\boldsymbol{M}_{\mathrm{T}}^{-1})_{22} - T_{i11}]\,b - T_{i21}\,c = -(\boldsymbol{M}_i\boldsymbol{M}_{\mathrm{T}}^{-1})_{21}, \tag{7}$$

$$[(\boldsymbol{M}_i\boldsymbol{M}_{\mathrm{T}}^{-1})_{11} - T_{i22}]\,a + (\boldsymbol{M}_i\boldsymbol{M}_{\mathrm{T}}^{-1})_{12}\,c = T_{i12}, \tag{8}$$

and

$$(M_t M_T^{-1})_{21}\, a - T_{t12}\, b + [(M_t M_T^{-1})_{22} - T_{t22}]\, c = 0. \tag{9}$$

One-port standards place the two constraints

$$a - \Gamma_t \Gamma_{t1}\, b - \Gamma_{t1}\, c = -\Gamma_t \tag{10}$$

and

$$\Gamma_t[\Gamma_{t2}(M_T^{-1})_{11} - (M_T^{-1})_{21}]a + [\Gamma_{t2}(M_T^{-1})_{12} - (M_T^{-1})_{22}]b + \Gamma_t[\Gamma_{t2}(M_T^{-1})_{12} - (M_T^{-1})_{22}]c = (M_T^{-1})_{21} - \Gamma_{t2}(M_T^{-1})_{11} \tag{11}$$

on the unknowns a, b, and c, where Γ_t is the actual reflection coefficient of the one-port, Γ_{t1} is its measured reflection coefficient when it is connected to the first port of the analyzer, and Γ_{t2} is its measured reflection coefficient when it is connected to the second port of the analyzer. The reference impedance of the matrices T_t and reflection coefficients Γ_{t1} and Γ_{t2} must be consistent and set the reference impedance of the calibration.

When enough reflective and transmissive standards have been measured, these equations will overdetermine a, b, and c. There are many ways to solve for these overdetermined parameters; we used a conventional least-squares fitting algorithm [3]. This approach averages errors and avoids numerical problems due to the linear dependence of the unknowns in (6), (7), (8), and (9).

III. CALIBRATION ARTIFACTS

We fabricated CPW standards on a gallium arsenide substrate to support both TRL and series-resistor calibrations. The CPWs were made by evaporating a 50 nm thick adhesion layer of titanium followed by a 500 nm thick gold film onto a 500 µm thick gallium arsenide substrate; the center conductor was 64 µm wide and was separated from two 261.5 µm wide ground planes by 42 µm gaps. The TRL CPW standards consisted of a thru line 0.550 mm long, five longer lines of length 2.685 mm, 3.750 mm, 7.115 mm, 20.245 mm, and 40.550 mm, and symmetric shorts offset 0.225 mm from the beginning of the CPW. The series-resistor standards consisted of a 128 µm long by 64 µm wide nickel-chromium thin-film resistor placed in the center of a 0.550 mm long CPW line.

IV. RESISTOR MODELS

The calibration requires that we know the actual transmission matrix T_t for each two-port standard. We assessed the accuracy of the series-resistor calibration using two different models to generate the series-resistor's matrix T_t. Figure 1 shows the two series-resistor models. The first model in Fig. 1(a) assumes that the series-resistor's impedance is just its measured dc resistance R_{dc}.

The second series-resistor model in Fig. 1(b) takes into account some of the circuit parasitics and the resistor's distributed nature. Here the embedding circuit is modeled as a transmission line with a gap in the middle of the center conductor, as shown in the test circuit of Fig. 1(c). C_c represents the capacitance across the gap in the center conductor, while C_e is the end (or fringing) capacitance of each center conductor to ground. When R_{dc}=0 the model reduces to a thru for small Δl.

We determined C_c and C_e by measuring the S-parameters of the test circuit of Fig. 1(c) with a multiline TRL calibration. This measurement needs to be performed only once for a particular waveguide geometry. For our CPW and series-resistor circuit geometry, we obtained C_e=4.8 fF and C_c=1.7 fF. We determined the capacitance C_0 per unit length of CPW from a measurement of a matched load using the method of [4]. We then set $\Delta l = C_e/C_0$ so that when $R_{dc}=\infty$ the model agrees with the open circuit test structure of Fig. 1(c). Once C_c and Δl are set in the model, the only additional measurement required for each individual series-resistor standard is its dc resistance R_{dc}, which is easily measured.

V. ACCURACY

We assessed the accuracy of our series-resistor calibrations by comparing them to a multiline TRL probe-tip calibration using the method of [5], which determines the upper bound for $|S'_{ij} - S_{ij}|$, where S'_{ij} are the S-parameters of any passive device measured by the series-resistor calibration and S_{ij} are the S-parameters measured by the TRL calibration. The bounds are valid for $ij \in \{11, 21, 12, 22\}$ when $|S_{11}| \le 1$, $|S_{22}| \le 1$ and $|S_{12}\,S_{21}| \le 1$. We set the reference plane of each calibration at the center of the thru and the reference impedance to 50 Ω.

Figure 2 shows the measurement error bound for a typical series-resistor calibration as the curve marked with circles compared to our 50 Ω TRL reference calibration. Here R_{dc}= 223.7 Ω and we used the dc resistor model of Fig. 1(a); the bounds for similar calibrations using series resistors in the range of 51 Ω to 267 Ω were comparable to that shown in the figure. The curve marked with triangles is the measurement error bound for a representative LRM calibration that also models the match standards as a dc resistance: the error bound is comparable.

Also shown for comparison as a dashed curve is the measurement error bound due to instrument drift, calculated from TRL calibrations performed at the beginning and end of the experiment. We found that the series-resistor calibration error bound is much higher than that due to instrument drift, which suggests that the series-resistor calibration might be improved by using a better electrical model for the resistor element.

The curve in Fig. 2 marked with dots shows the improvement in the measurement error bound upon application of the distributed series-resistor model of Fig. 1(b): using this model reduces the error bound by approximately a factor of two relative to the bound using the dc model. We observed a similar improvement for the other resistors investigated in the experiment upon applying the distributed model.

VI. CONCLUSIONS

We used a simple but general algorithm to investigate an automatic network analyzer calibration based on a thru, a reflect, and a series resistor that is sufficiently accurate for many MMIC applications. The accuracy of a typical series-resistor calibration is comparable to an LRM calibration in CPW when we used a simple dc model for the resistor standards. A distributed model improves the accuracy of the series-resistor calibration substantially. This improved model can be implemented in a compact on-wafer series-resistor calibration set with the addition of only a simple series-open circuit.

ACKNOWLEDGMENT

The authors thank Nita Morgan for skillful fabrication of the CPW test wafers used in this work.

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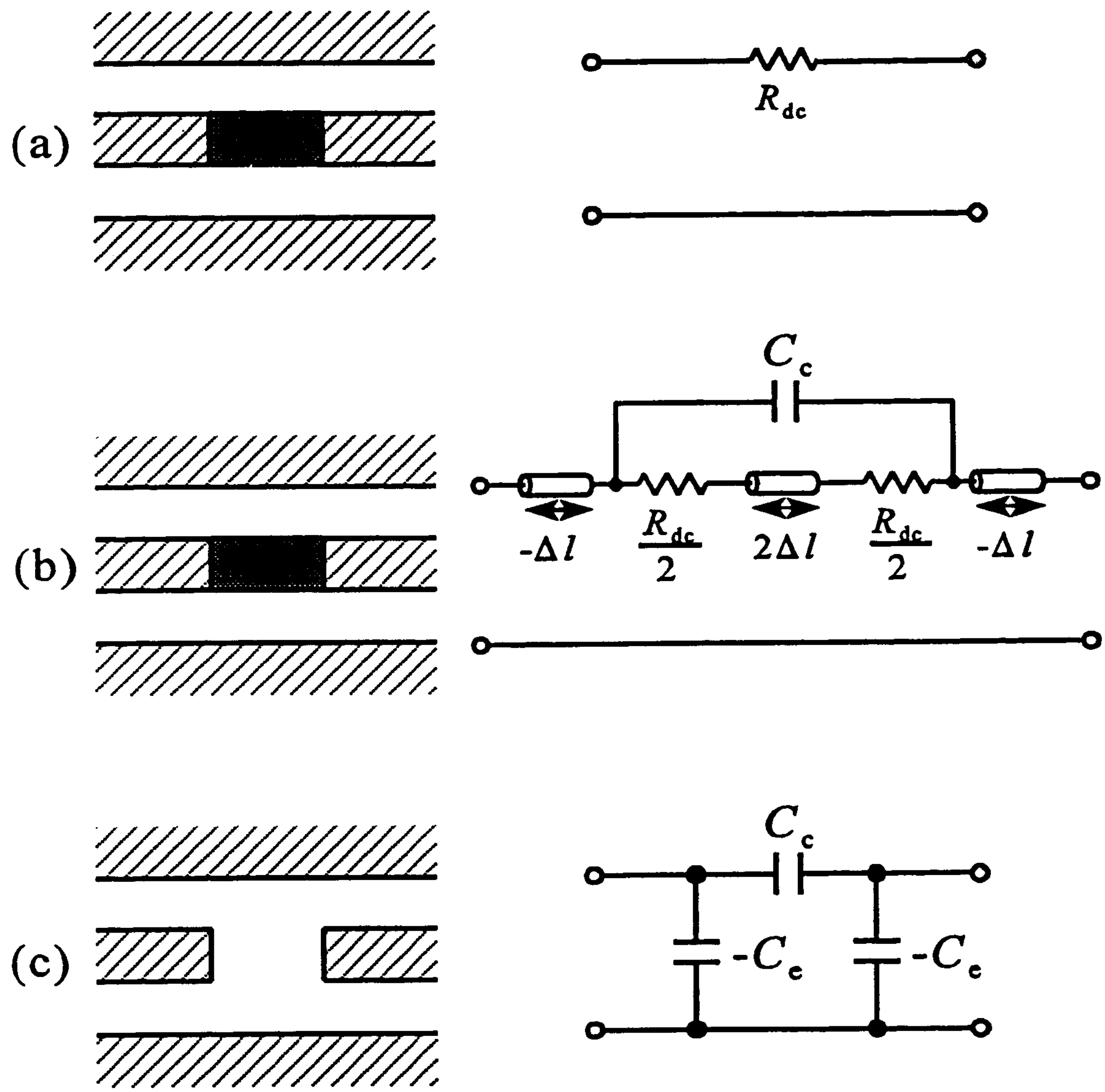

Fig. 1. Series-resistor circuit and models. The measurement reference plane is at the center of the circuit. (a) series-resistor standard and simple model; (b) series-resistor standard and distributed model; (c) circuit used to measure and determine C_c and C_e.

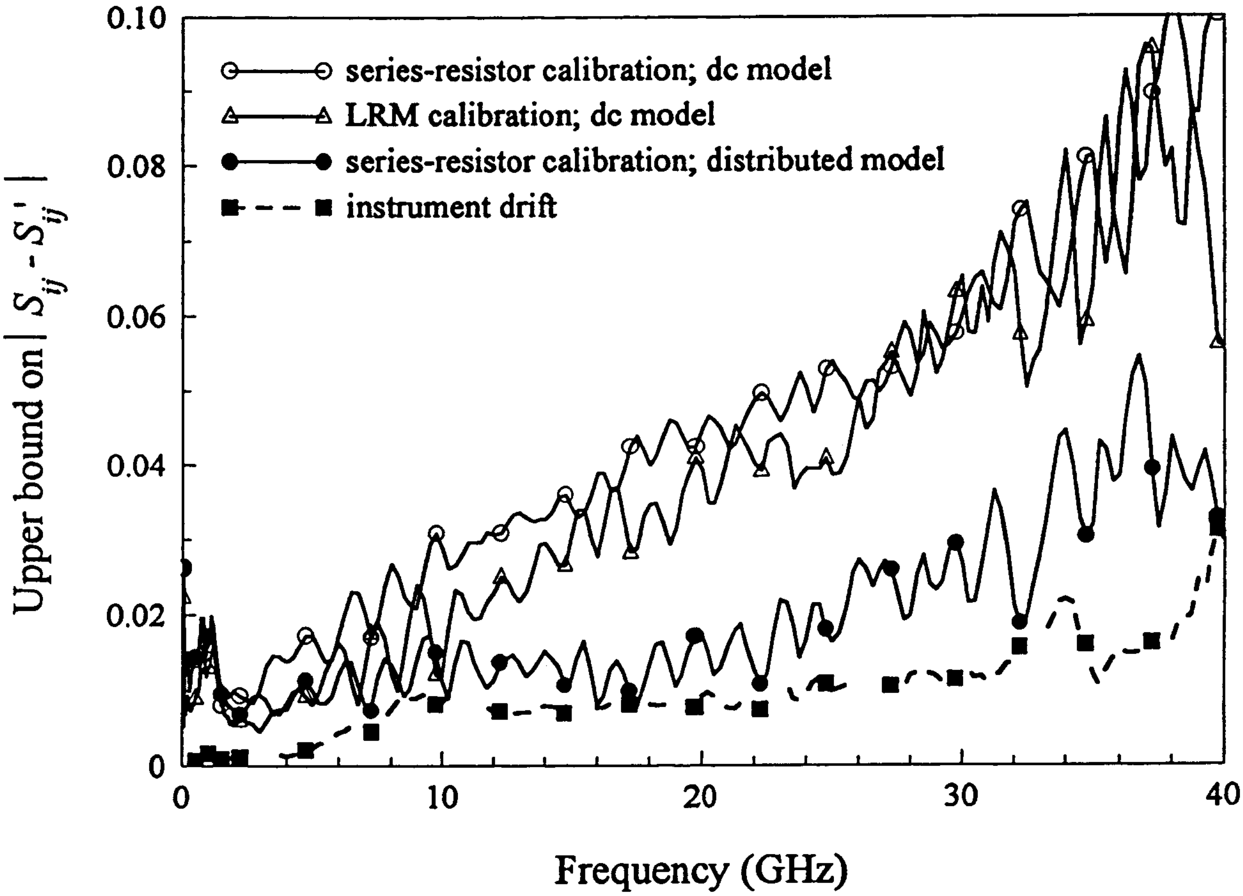

Fig. 2. Measurement error bounds for two implementations of the series-resistor calibration and a conventional LRM calibration relative to a TRL calibration. The error bound due to test set drift and contact errors is shown for comparison.